

Lecture 5
2025/2026

Microwave Devices and Circuits for Radiocommunications

2025/2026

- 2C/1L, **MDCR**
- Attendance at minimum 7 sessions (course or laboratory)
- Lectures- **associate professor Radu Damian**
 - Tuesday **12-14, P2**
 - E – 50% final grade
 - problems + (2p atten. lect.) + (3 tests) + (bonus activity)
 - first test L1: 24.02.2026 (t2 and t3 not announced, lecture)
 - 3att.=+0.5p
 - all materials/equipments authorized

2025/2026

- Laboratory – **associate professor Radu Damian**
 - Monday 14-16, II.13 / (even weeks)
 - L – 25% final grade
 - ADS, 4 sessions
 - Attendance + **personal results**
 - P – 25% final grade
 - ADS, 3 sessions (-1? 24.02.2026)
 - personal homework

Lecture 6

- (Only) Lecture 6 - 24.03.2026
 - ~~12-14 P2~~
 - **16-18 P6**

Materials

■ <https://rf-opto.etti.tuiasi.ro>

The screenshot shows a web browser window with the URL https://rf-opto.etti.tuiasi.ro/microwave_cd.php?chg_lang=0. The page features a dark blue navigation bar with links for Main, Courses, Master, Staff, Research, Students, and Admin. Below this is a secondary navigation bar with links for Microwave CD, Optical Communications, Optoelectronics, Internet, Antennas, Practica, Networks, and Educational software. The main content area is titled "Microwave Devices and Circuits for Radiocommunications (English)" and includes the following information:

- Course: MDCR (2017-2018)**
- Course Coordinator:** Assoc.P. Dr. Radu-Florin Damian
- Code:** EDOS412T
- Discipline Type:** DOS; Alternative, Specialty
- Credits:** 4
- Enrollment Year:** 4, Sem. 7

Activities

Course: Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:
Laboratory: Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

Evaluation

Type: **Examen**

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

[Aggregate Results](#)

Attendance

[Course](#)
[Laboratory](#)

Lists

[Bonus-uri acumulate \(final\)](#)
[Studenti care nu pot intra in examen](#)

Materials

Course Slides

[MDCR Lecture 1](#) (pdf, 5.43 MB, en, [↗](#))
[MDCR Lecture 2](#) (pdf, 3.67 MB, en, [↗](#))
[MDCR Lecture 3](#) (pdf, 4.76 MB, en, [↗](#))
[MDCR Lecture 4](#) (pdf, 5.58 MB, en, [↗](#))

On the right side of the screenshot, there is a dark blue banner for "RF-OPTO" with the ETTI logo and the University of Tuiasi logo. The banner includes a language selection menu with "English" (circled in red) and "Romana". Below the banner is another navigation bar with links for Main, Courses, Master, Staff, and Research. A secondary navigation bar below that has links for Grades, Student List, Exams, and Photos. The main content area on the right is titled "Online Exams" and includes the text "In order to participate at online exams you must get ready following".

Materials

- RF-OPTO
 - <https://rf-opto.etti.tuiasi.ro>
- **David Pozar, “Microwave Engineering”,**
Wiley; 4th edition , 2011
 - 1 exam problem ← Pozar
- Photos
 - sent by **online exam**
 - used at lectures/laboratory

Examen: Logarithmic scales

$$\text{dB} = 10 \cdot \log_{10} (P_2 / P_1)$$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

$$\text{dBm} = 10 \cdot \log_{10} (P / 1 \text{ mW})$$

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 μ W
-30 dBm	= 1 μ W
-60 dBm	= 1 nW

$$[\text{dBm}] + [\text{dB}] = [\text{dBm}]$$

$$[\text{dBm/Hz}] + [\text{dB}] = [\text{dBm/Hz}]$$

$$[\text{x}] + [\text{dB}] = [\text{x}]$$

Exam

- Complex numbers arithmetic!!!!
- $z = a + j \cdot b ; j^2 = -1$

Introduction

~ Microwaves

- Electrical Length (Phase Length)
 - l – physical length
 - $E = \beta \cdot l$ – electrical Length

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$

$$E = \beta \cdot l = \frac{2\pi}{c_0} \cdot (l \cdot f \cdot \sqrt{\epsilon_r})$$

V, I vary
~ useless

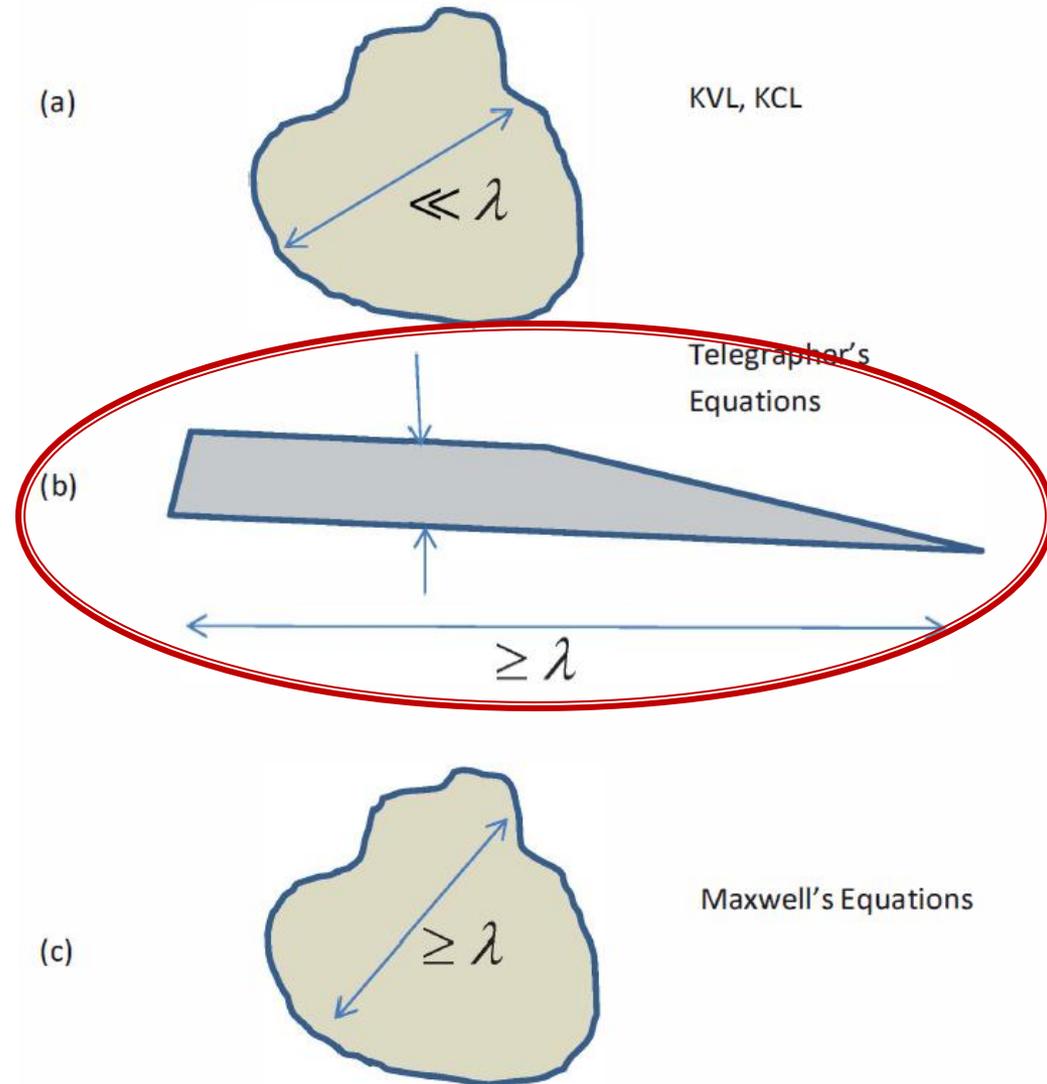
- Dependency
 - antenna gain
 - radar cross-section

Electrical Length

- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest

- $E \approx 0 \rightarrow$ Kirchhoff
- $E > 0 \rightarrow$ wave propagation

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda}\right)$$

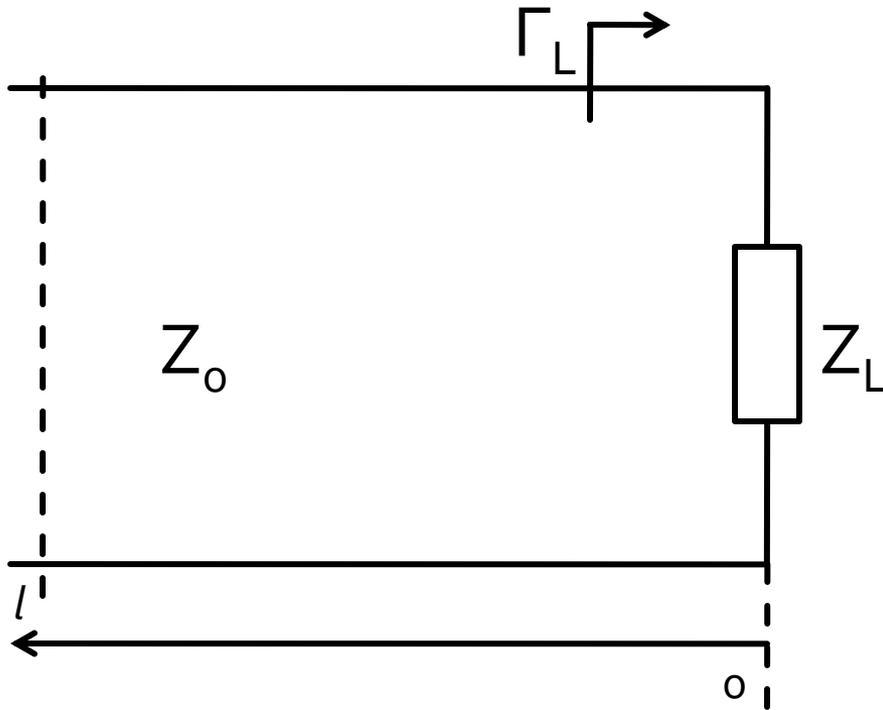


TEM transmission lines

Course Topics

- **Transmission lines**
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers?~~

The lossless line



$$V(z) = V_0^+ e^{-j\beta \cdot z} + V_0^- e^{j\beta \cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta \cdot z} - \frac{V_0^-}{Z_0} e^{j\beta \cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \quad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Z_0 real

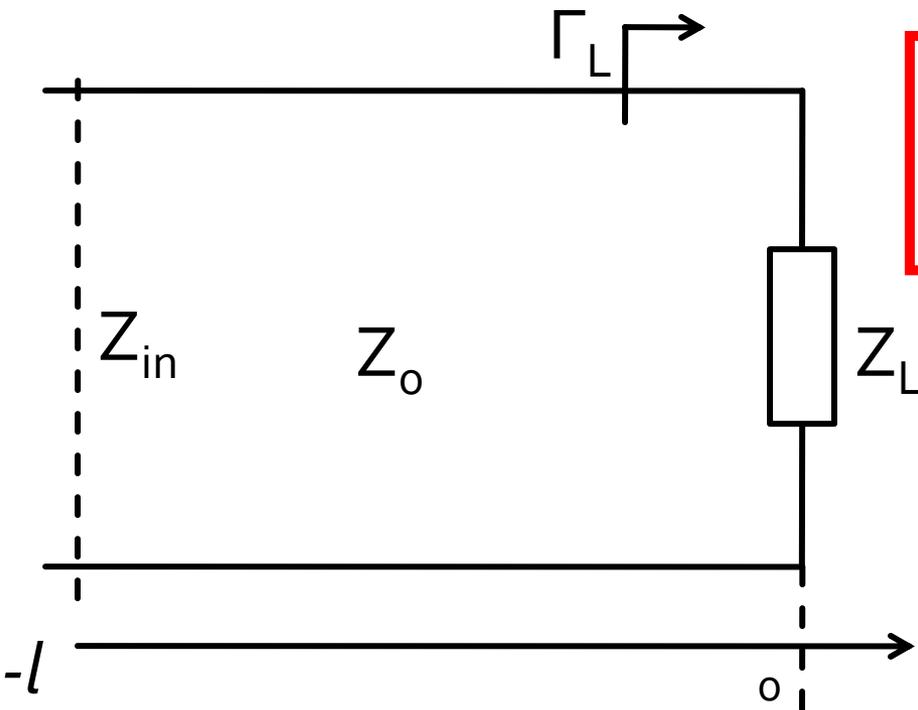
The lossless line

$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot (1 - |\Gamma|^2)$$

- Average power flow is constant along the line
 - (**no** $P_{avg}(\mathbf{z})$)
 - can be measured
- We can use the power to characterize the amplitude of a signal
 - a very “energetic” (basic physics) point of view
 - more power = “more” signal

The lossless line

- input impedance of a length l of transmission line with characteristic impedance Z_0 , loaded with an arbitrary impedance Z_L



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Power transfer

Impedance Matching

Matching , from the point of view of power transmission

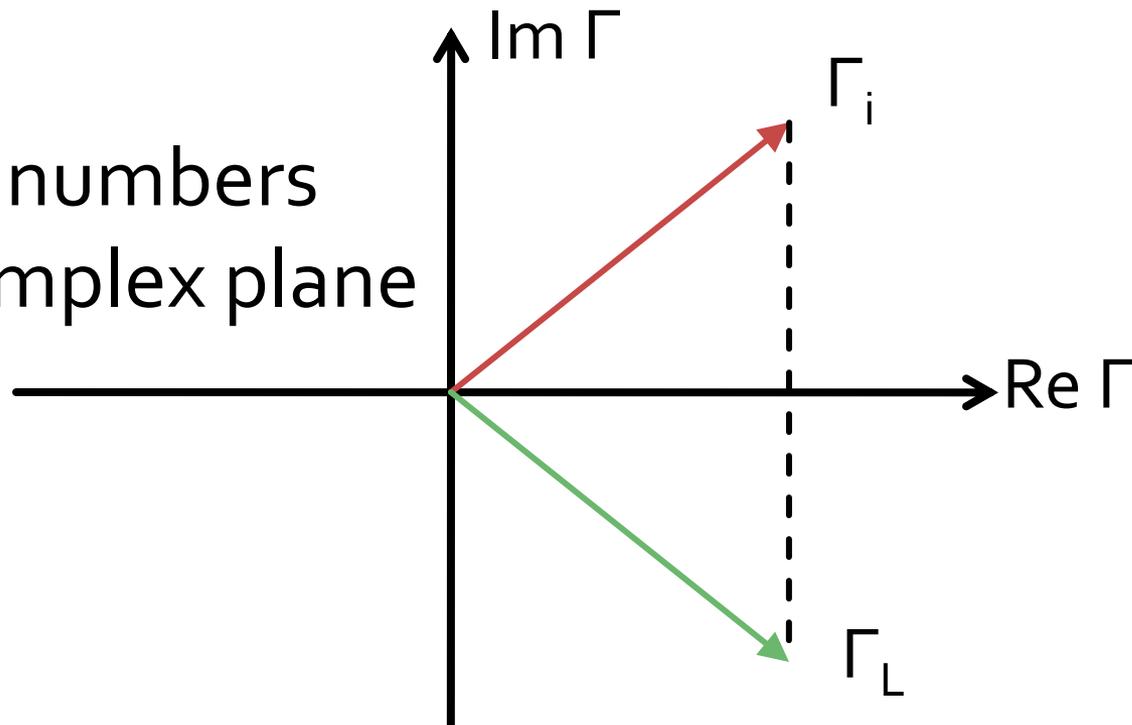
$$Z_L = Z_i^*$$

If we choose a (any) real Z_0

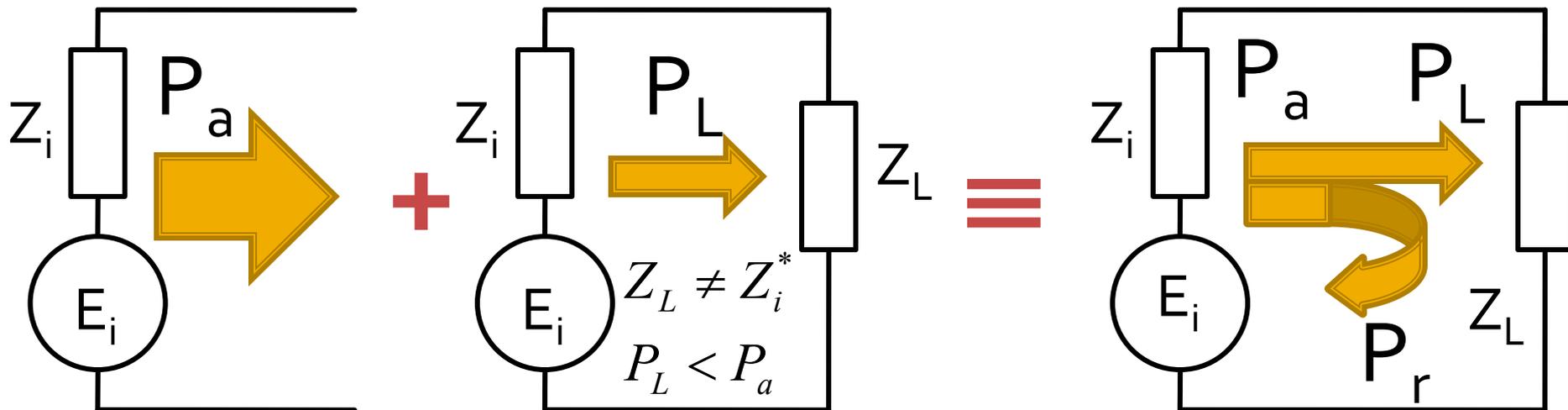
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane



Reflection and power / Model



- The source has the ability to send to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a - P_L$
- The power is a **scalar** !

The quarter-wave transformer

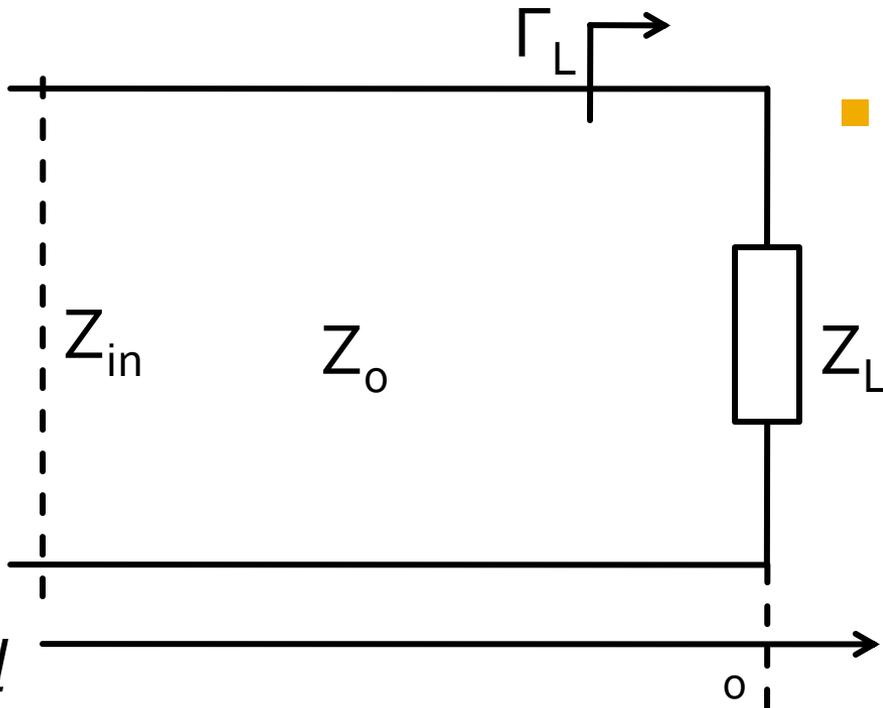
Impedance Matching

The lossless line, special cases

- $l = k \cdot \lambda/2$ $\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi$ $\tan \beta \cdot l = 0$
- $l = \lambda/4 + k \cdot \lambda/2$ $\beta \cdot l = \frac{\pi}{2} + k \cdot \pi$ $\tan \beta \cdot l \rightarrow \infty$

$$Z_{in} = Z_L$$

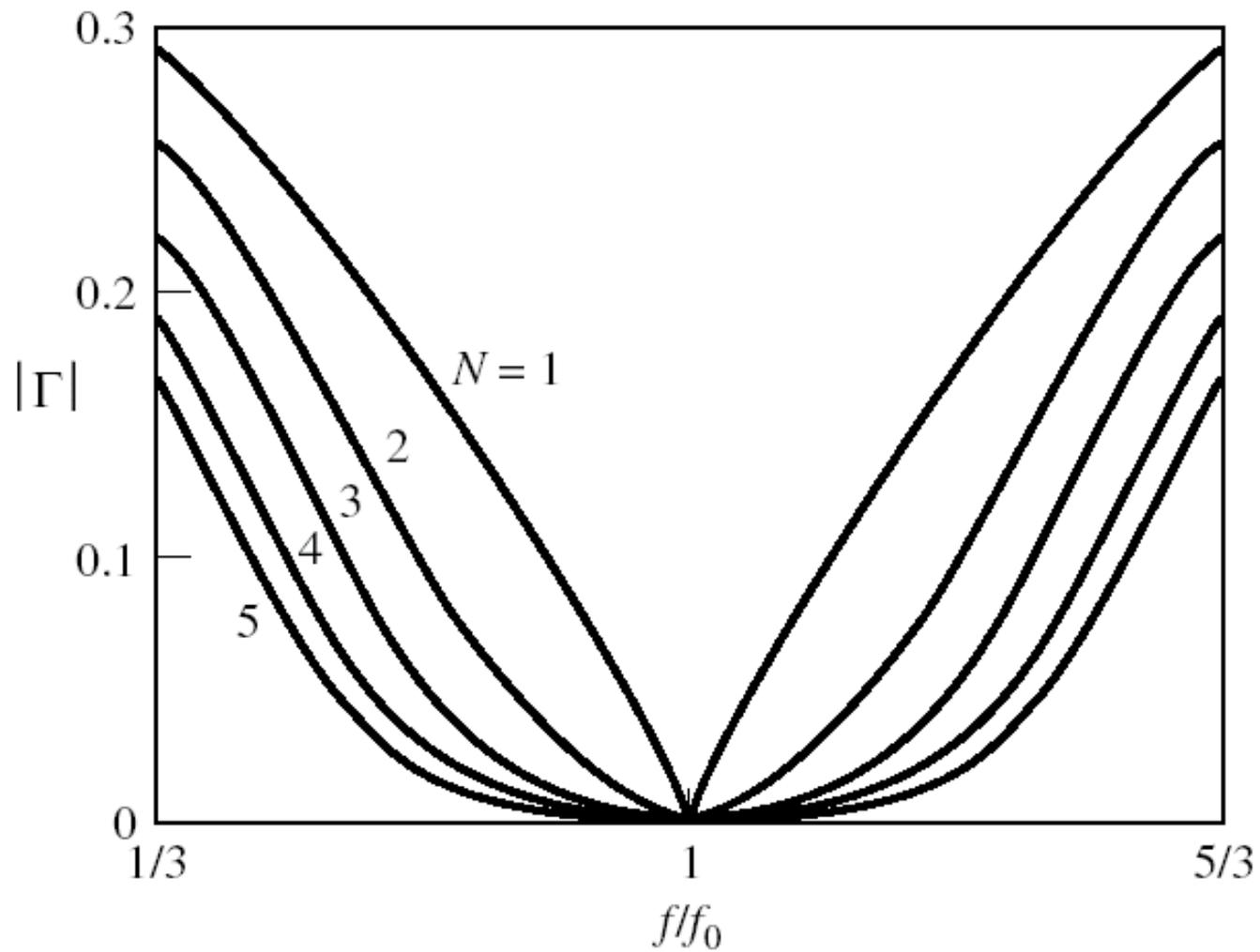
$$Z_{in} = \frac{Z_0^2}{Z_L}$$



- quarter-wave transformer

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Bandwidth



Binomial multisection transformer

Exact results

Z_L/Z_0	$N = 2$		$N = 3$			$N = 4$			
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

Z_L/Z_0	$N = 5$					$N = 6$					
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

Chebyshev polynomials

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

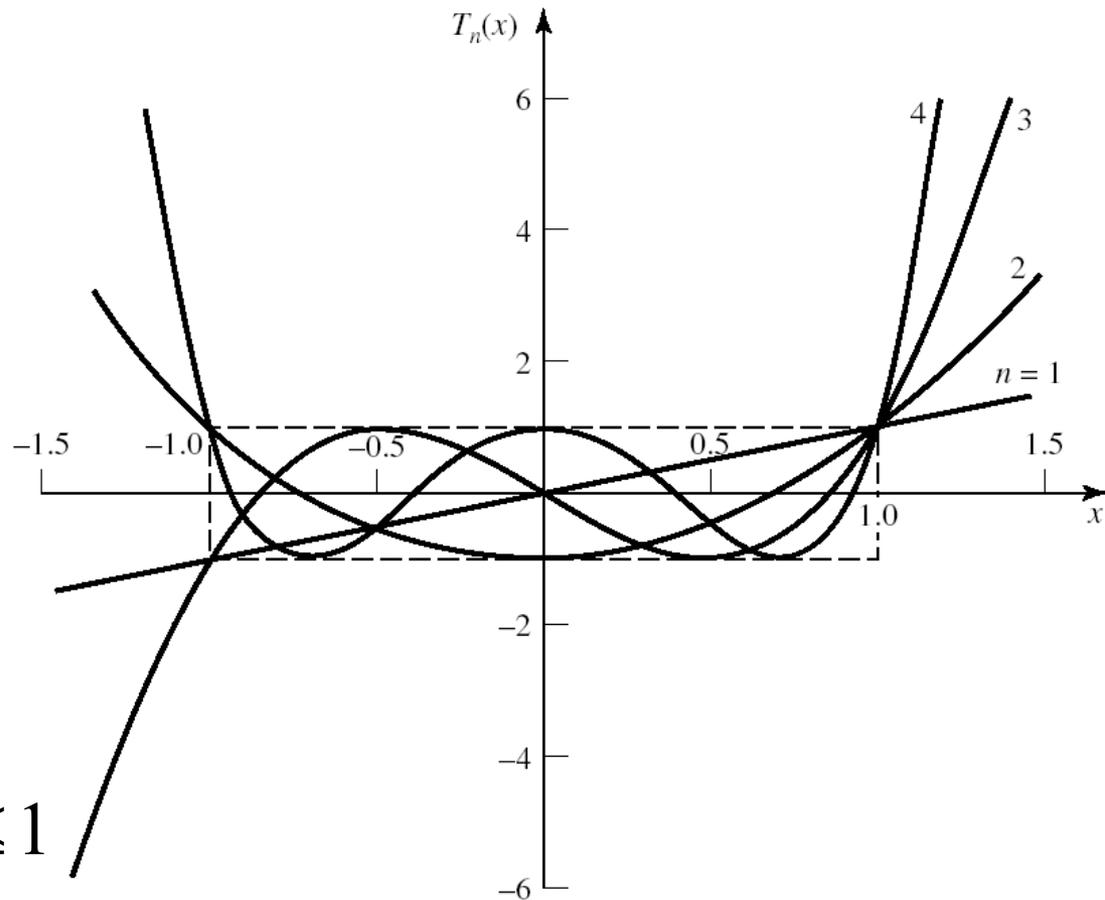
$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

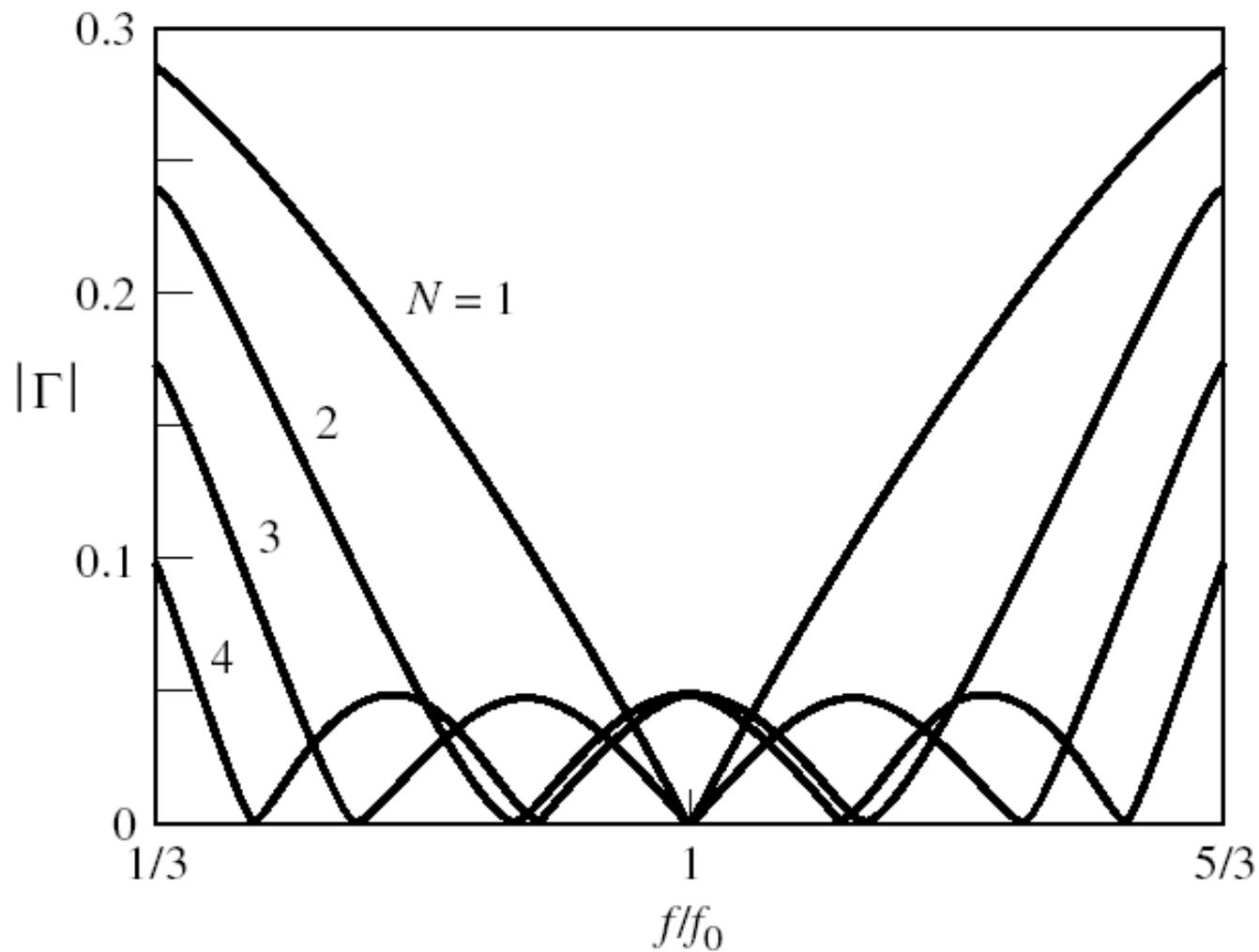
■ equal-ripple

$$-1 \leq x \leq 1 \Rightarrow |T_n(x)| \leq 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$



Bandwidth



Chebyshev multisection transformer

Exact results

Z_L/Z_0	$N = 2$				$N = 3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

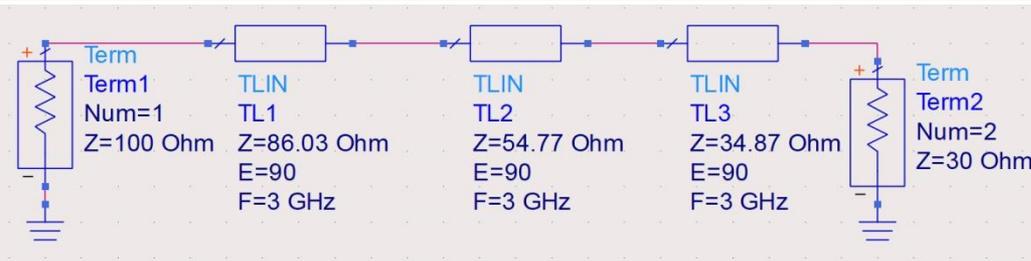
$N = 4$

Z_L/Z_0	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$			
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

Laboratory 1

Impedance Matching

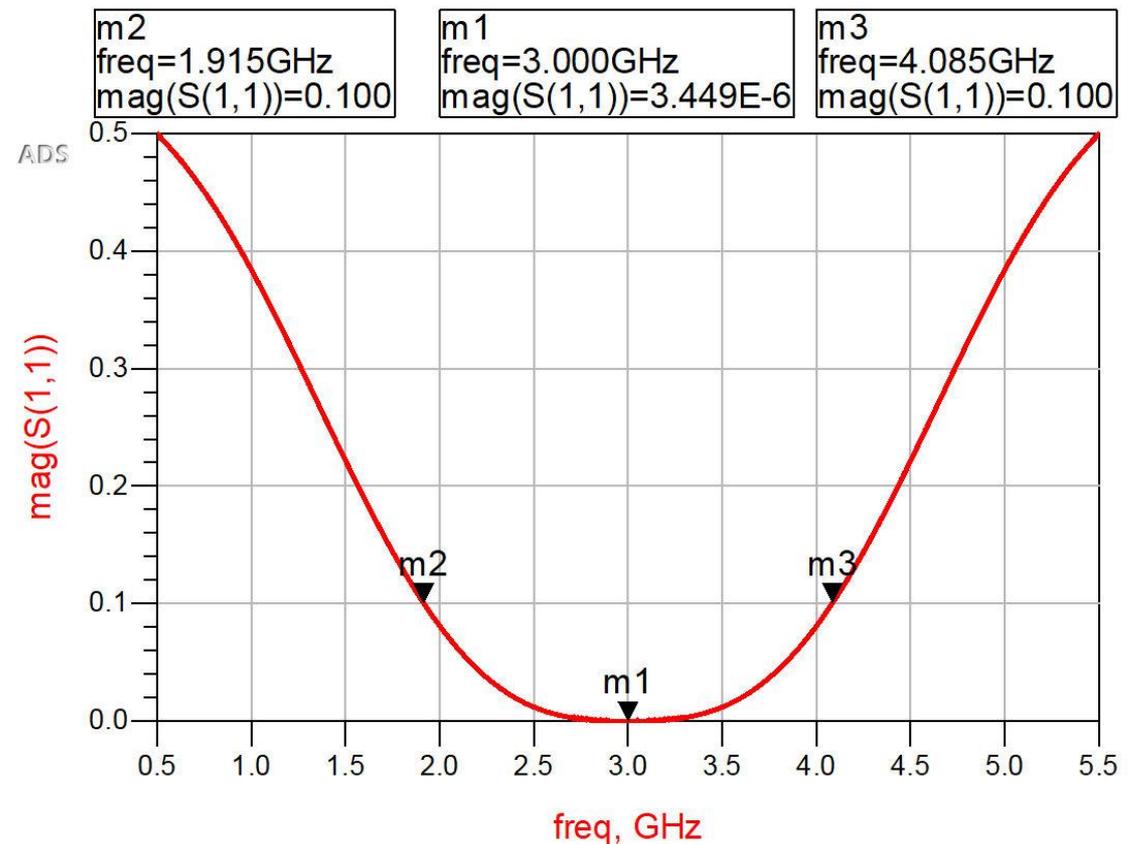
Binomial multisection transformer



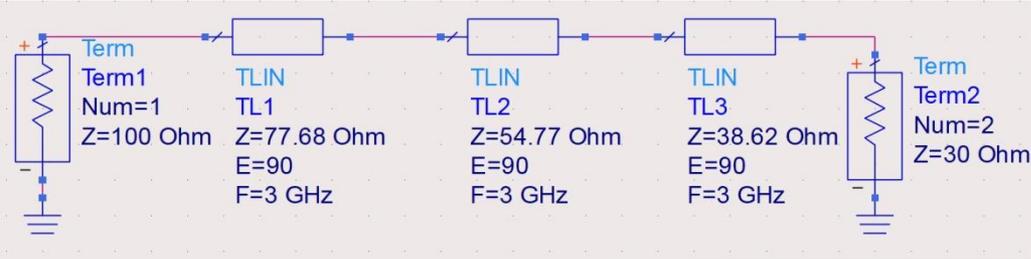
 S-PARAMETERS

S_Param
SP1
Start=0.5 GHz
Stop=5.5 GHz
Step=0.001 GHz

$$\Delta f = 2.169 \text{ GHz}$$

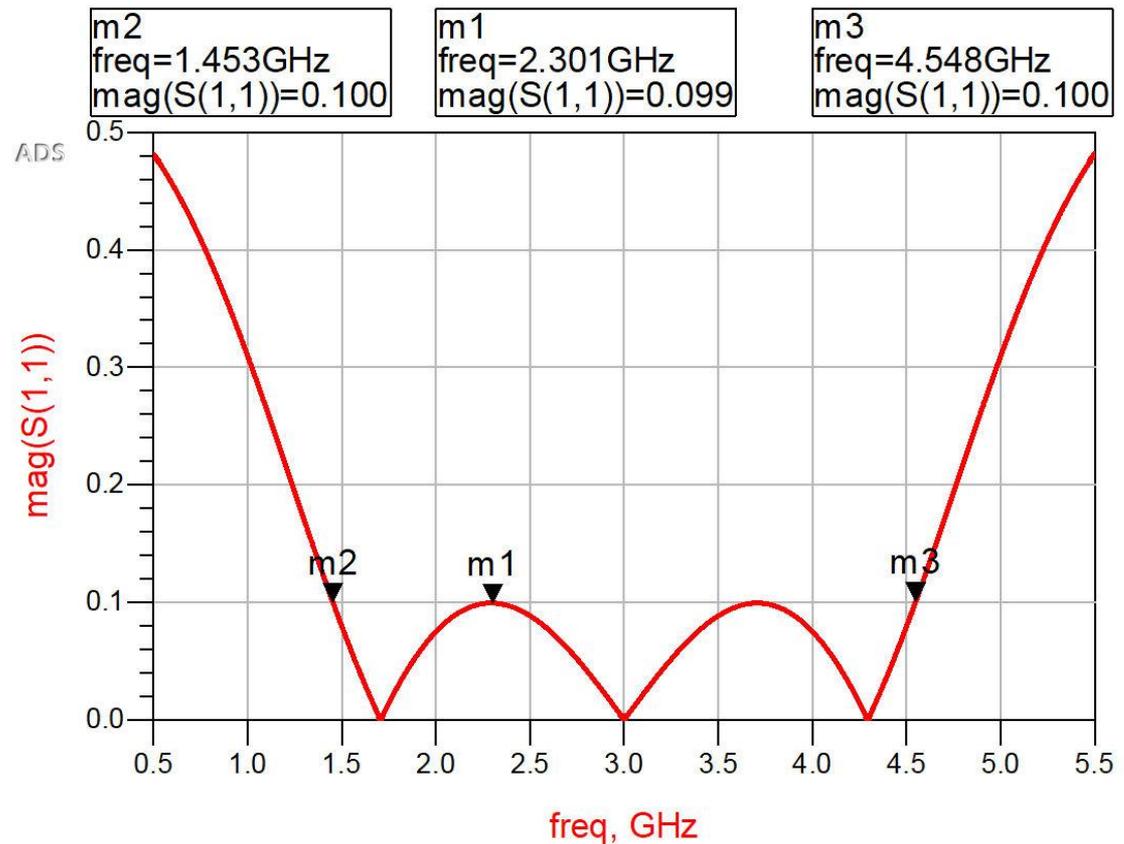


Chebyshev multisection transformer



 S-PARAMETERS

S_Param
SP1
Start=0.5 GHz
Stop=5.5 GHz
Step=0.001 GHz



$$\Delta f = 3.096 \text{ GHz}$$

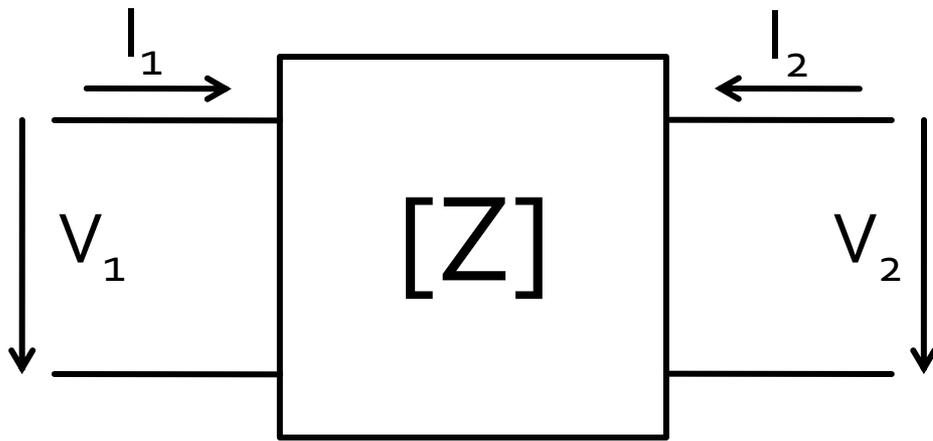
General theory

Microwave Network Analysis

Course Topics

- Transmission lines
 - Impedance matching and tuning
 - Directional couplers
 - Power dividers
 - Microwave amplifier design
 - Microwave filters
 - ~~Oscillators and mixers?~~
- 

Impedance matrix – Z



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$

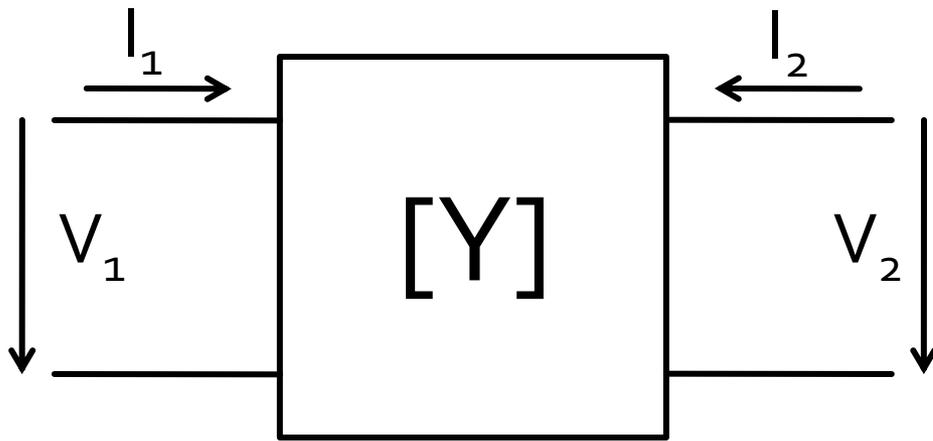
$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

$$V_1 = Z_{11} \cdot I_1 \Big|_{I_2=0} \quad Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

- Z_{11} – input impedance with open-circuited output

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} \quad Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} \quad Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} \quad Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

Admittance matrix – Y



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2$$

$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2$$

$$I_1 = Y_{11} \cdot V_1 \Big|_{V_2=0} \quad Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

- Y_{11} – input admittance with short-circuited output

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} \quad Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} \quad Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} \quad Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

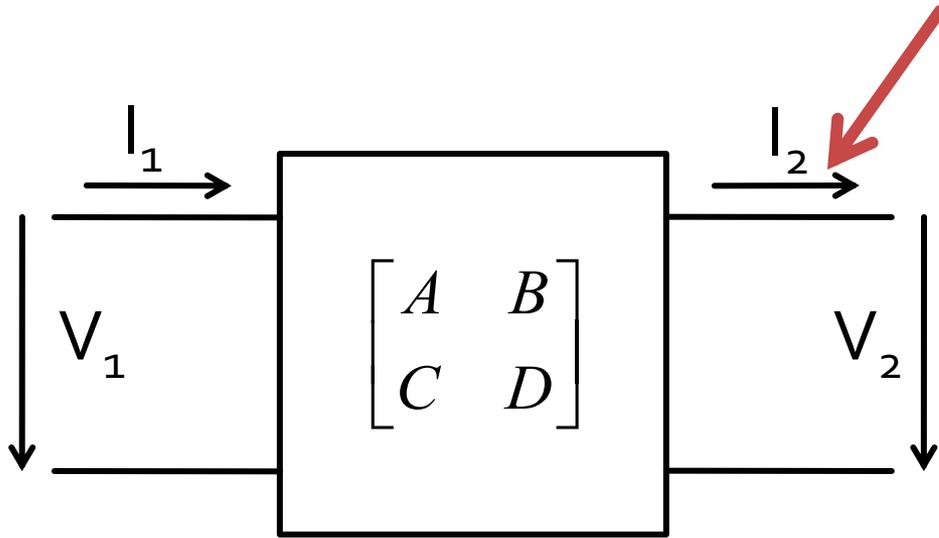
Network Analysis

- Each matrix is best suited for a particular mode of port excitation (V, I)
 - matrix H in common emitter connection for TB: I_B, V_{CE}
 - matrices provide the associated quantities depending on the “attack” ones
- Traditional notation of Z, Y, G, H parameters is in lowercase (z, y, g, h)
- In microwave analysis we prefer the notation in uppercase to avoid confusion with the **normalized parameters**

$$z = \frac{Z}{Z_0} \quad y = \frac{Y}{Y_0} = \frac{1/Z}{1/Z_0} = \frac{Z_0}{Z} = Z_0 \cdot Y$$

$$z_{11} = \frac{Z_{11}}{Z_0} \quad y_{11} = \frac{Y_{11}}{Y_0} = Z_0 \cdot Y_{11}$$

ABCD (transmission) matrix



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

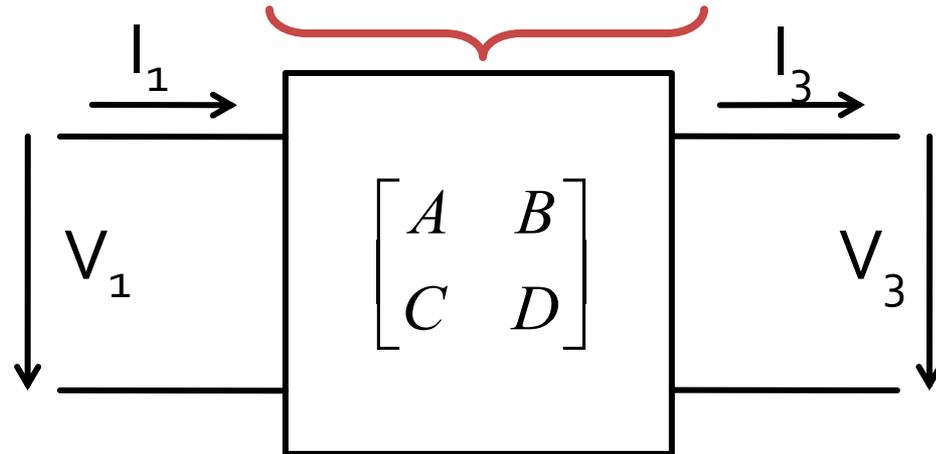
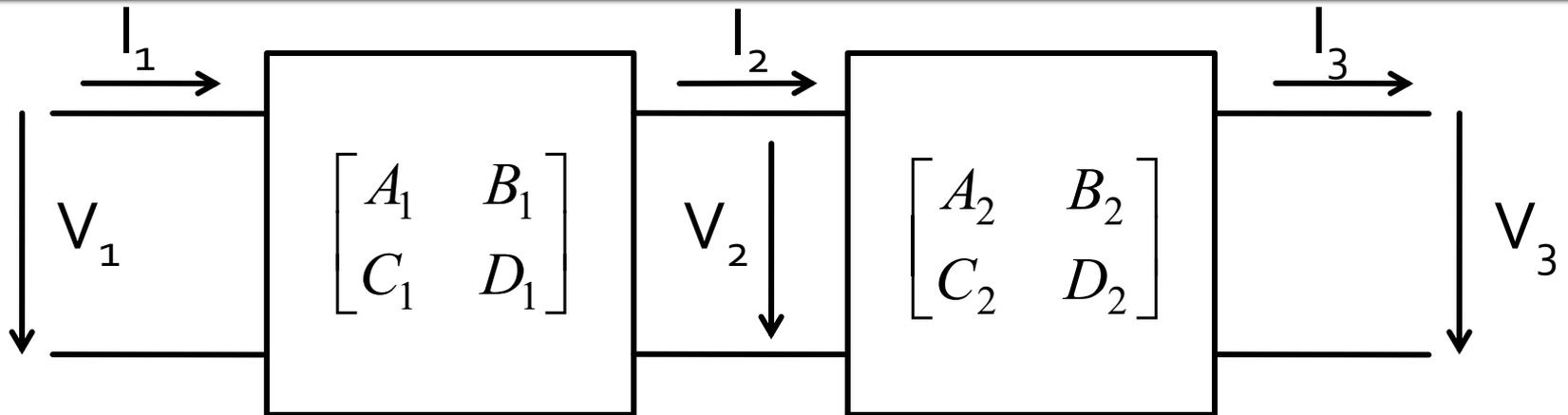
$$V_1 = A \cdot V_2 + B \cdot I_2$$

$$I_1 = C \cdot V_2 + D \cdot I_2$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \frac{1}{A \cdot D - B \cdot C} \cdot \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \quad B = \left. \frac{V_1}{I_2} \right|_{V_2=0} \quad C = \left. \frac{I_1}{V_2} \right|_{I_2=0} \quad D = \left. \frac{I_1}{I_2} \right|_{V_2=0}$$

ABCD (transmission) matrix



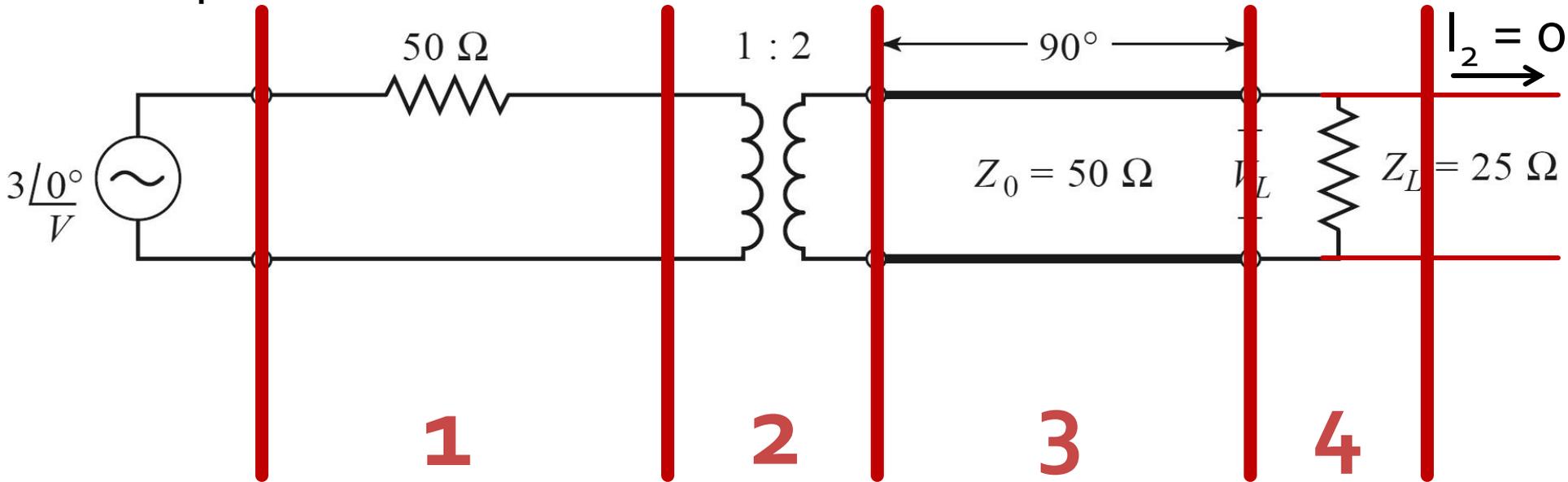
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

ABCD (transmission) matrix

- suitable **only** for two-port networks (Z, Y can be easily extended for multiport / n -ports)
- allows easy coupling of multiple elements
- allows the calculation of complex circuits with one input and one output by breaking them in individual component blocks
- a library of ABCD matrices for elementary two-port networks can be built up

Example for ABCD matrix

- We break the circuit in elementary sections
- Sources are left outside
- If necessary, input and output ports are created (and left open-circuited)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = M_1 \cdot M_2 \cdot M_3 \cdot M_4 \quad V_1 = A \cdot V_2 + B \cdot I_2 \Big|_{I_2=0} \quad V = A \cdot V_L \rightarrow V_L = \frac{V}{A}$$

Homework

Subject no. 1

MDCR Lecture 4 (Impedance Matching / Network Analysis) – supplemental activity

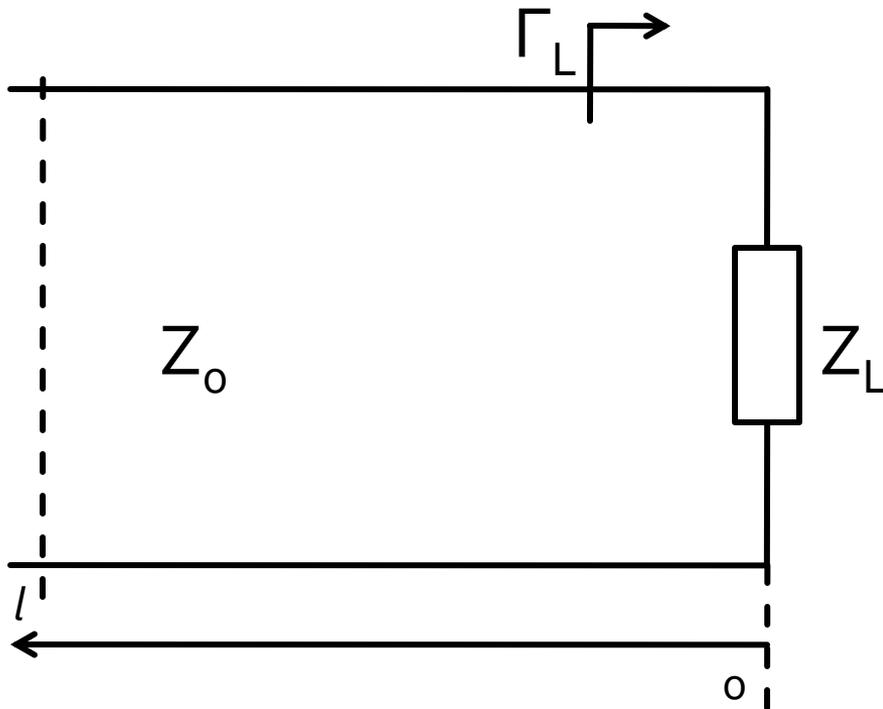
Write on a piece of paper and scan (or photograph, use app, etc. to obtain 1 pdf/jpg file) the proofs/relationships:

- for finding the expression for the 4th order Chebyshev polynomial (similar to L4 S69).
- for finding the ABCD matrix for (similar to L4 S96 for S97-101):
 - shunt admittance
 - transmission line
 - transformer
 - π network
 - T network

(Somewhat!) Specific theory

Microwave Network Analysis

The lossless line



$$V(z) = V_0^+ e^{-j\beta \cdot z} + V_0^- e^{j\beta \cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta \cdot z} - \frac{V_0^-}{Z_0} e^{j\beta \cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \quad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Z_0 real

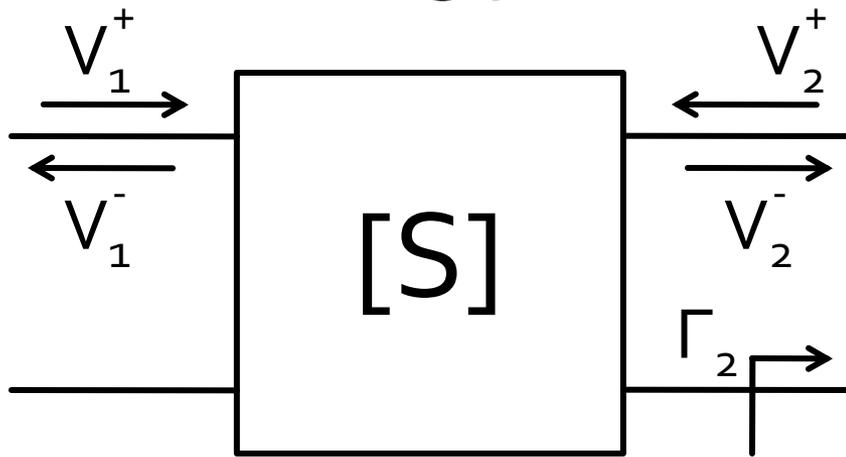
The lossless line

$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot (1 - |\Gamma|^2)$$

- Average power flow is constant along the line
 - (**no** $P_{avg}(\mathbf{z})$)
 - can be measured
- We can use the power to characterize the amplitude of a signal
 - a very “energetic” (basic physics) point of view
 - more power = “more” signal

Scattering matrix – S

- Scattering parameters



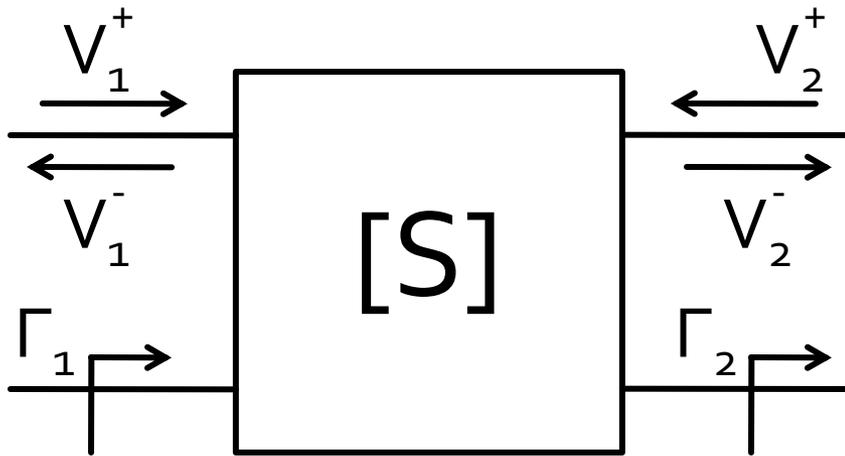
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0} \quad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Scattering matrix – S



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} = \Gamma_1 \Big|_{\Gamma_2=0}$$

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0} = T_{21} \Big|_{\Gamma_2=0}$$

- S_{11} is the reflection coefficient seen looking into port **1** when port **2** is terminated in matched load
- S_{21} is the transmission coefficient from port **1** (**second** index!) to port **2** (**first** index!) when port **2** is terminated in matched load

Scattering matrix – S

- S matrix can be extended to multiple ports

$$S_{ii} = \left. \frac{V_i^-}{V_i^+} \right|_{V_k^+ = 0, \forall k \neq i} \quad S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0, \forall k \neq j}$$

- S_{ii} is the reflection coefficient seen looking into port i when all other ports are terminated in matched loads
- S_{ij} is the transmission coefficient from port j (**second** index!) to port i (**first** index!) when all other ports are terminated in matched loads

Properties of S matrix

- If port i is connected to a transmission line with characteristic impedance Z_{0i}

$$[Z_0] = \begin{bmatrix} Z_{01} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Z_{0n} \end{bmatrix}$$

- Lecture 2 $V(z) = V_0^+ e^{-j\beta \cdot z} + V_0^- e^{j\beta \cdot z}$ $I(z) = \frac{V_0^+}{Z_0} e^{-j\beta \cdot z} - \frac{V_0^-}{Z_0} e^{j\beta \cdot z}$

In the port's reference plane, $z=0$

$$V_i = V_i^+ + V_i^- \quad I_i = \frac{V_i^+}{Z_{0i}} - \frac{V_i^-}{Z_{0i}}$$

- Relation to Z matrix

$$[Z] \cdot [I] = [V]$$

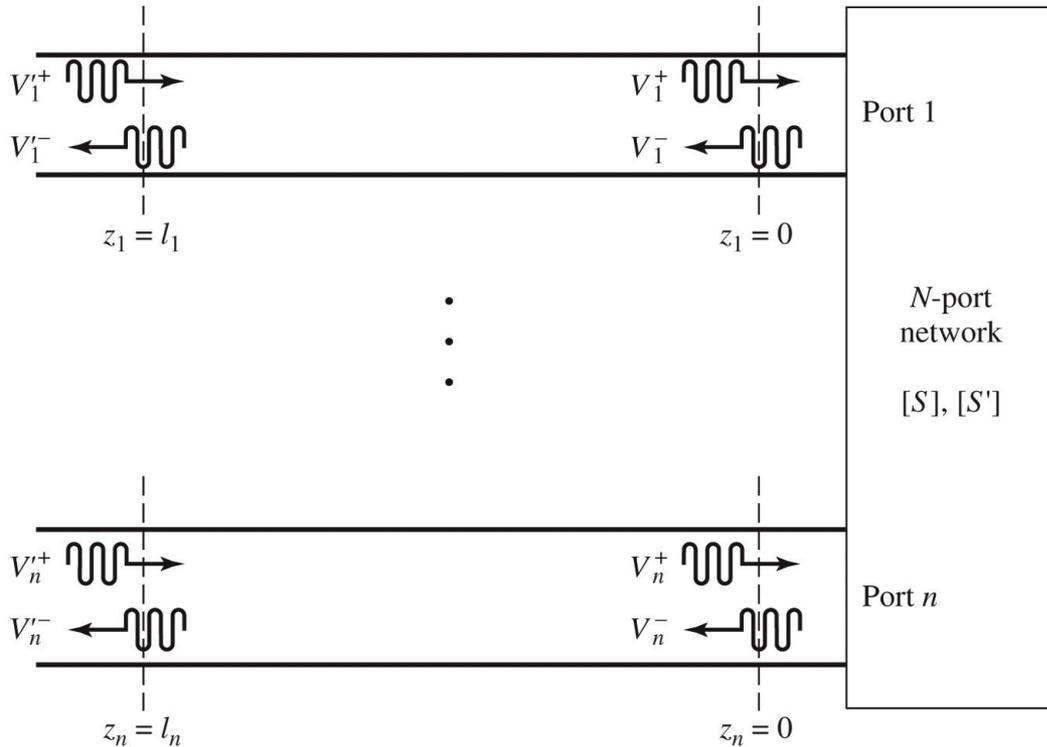
$$[Z] \cdot [I] = [Z_0]^{-1} \cdot [Z] \cdot [V^+] - [Z_0]^{-1} \cdot [Z] \cdot [V^-] \quad [V] = [V^+] + [V^-]$$

$$[Z_0]^{-1} \cdot [Z] \cdot [V^+] - [Z_0]^{-1} \cdot [Z] \cdot [V^-] = [V^+] + [V^-] \quad ([Z] - [Z_0]) \cdot [V^+] = ([Z] + [Z_0]) \cdot [V^-]$$

$$[V^-] = [S] \cdot [V^+]$$

$$[S] = ([Z] - [Z_0]) \cdot ([Z] + [Z_0])^{-1}$$

A Shift in Reference Planes



■ De-Embedding

Figure 4.9
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$$[S'] = \begin{bmatrix} e^{-j\theta_1} & 0 & \dots & 0 \\ 0 & e^{-j\theta_2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & e^{-j\theta_N} \end{bmatrix} \cdot [S] \cdot \begin{bmatrix} e^{-j\theta_1} & 0 & \dots & 0 \\ 0 & e^{-j\theta_2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & e^{-j\theta_N} \end{bmatrix}$$

Properties of S matrix (Z,Y)

- Reciprocal networks (no active circuits, no ferrites)

$$Z_{ij} = Z_{ji}, \forall j \neq i$$

$$Y_{ij} = Y_{ji}, \forall j \neq i$$

$$S_{ij} = S_{ji}, \forall j \neq i$$

$$[S] = [S]^t$$

- Lossless networks

$$\operatorname{Re}\{Z_{ij}\} = 0, \forall i, j$$

$$\operatorname{Re}\{Y_{ij}\} = 0, \forall i, j$$

$$\sum_{k=1}^N S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$[S]^* \cdot [S]^t = [1]$$

$$\sum_{k=1}^N S_{ki} \cdot S_{ki}^* = 1$$

$$\sum_{k=1}^N S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Generalized Scattering Parameters

- The total voltage and current on a transmission line in terms of the incident and reflected voltage wave amplitudes

$$V = V_0^+ + V_0^- \quad I = \frac{1}{Z_0} \cdot (V_0^+ - V_0^-) \quad \text{In the port's reference plane, } z=0$$

- We find the incident and reflected voltage wave amplitudes

$$V_0^+ = \frac{V + Z_0 \cdot I}{2} \quad V_0^- = \frac{V - Z_0 \cdot I}{2}$$

- The average power delivered to a load :

$$P_L = \frac{1}{2} \cdot \text{Re}\{V \cdot I^*\} = \frac{1}{2 \cdot Z_0} \cdot \text{Re}\left\{ |V_0^+|^2 - \underbrace{V_0^+ \cdot V_0^{-*} + V_0^{+*} \cdot V_0^-}_{(z - z^*) = \text{Im}} - |V_0^-|^2 \right\}$$

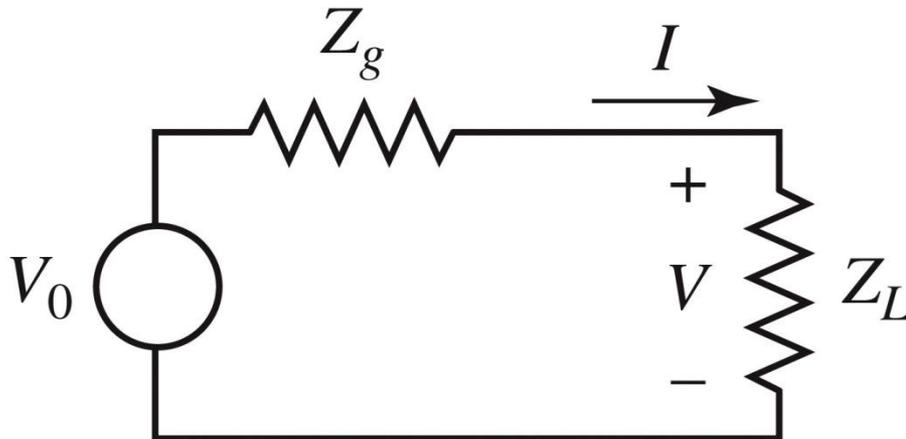
$$P_L = \frac{1}{2 \cdot Z_0} \cdot \left(|V_0^+|^2 - |V_0^-|^2 \right)$$

Generalized Scattering Parameters

- The average power delivered to a load:

$$P_L = \frac{1}{2 \cdot Z_0} \cdot \left(|V_0^+|^2 - |V_0^-|^2 \right)$$

- Restrictions
 - Result valid for Z_0 real
 - Requires the presence of a line with characteristic impedance Z_0 between the source and the load



Generalized Scattering Parameters

- We define the power wave amplitudes a and b

$$a = \frac{V + Z_R \cdot I}{2 \cdot \sqrt{R_R}} \quad \text{the incident power wave} \quad Z_R = R_R + j \cdot X_R$$

$$b = \frac{V - Z_R^* \cdot I}{2 \cdot \sqrt{R_R}} \quad \text{the reflected power wave}$$

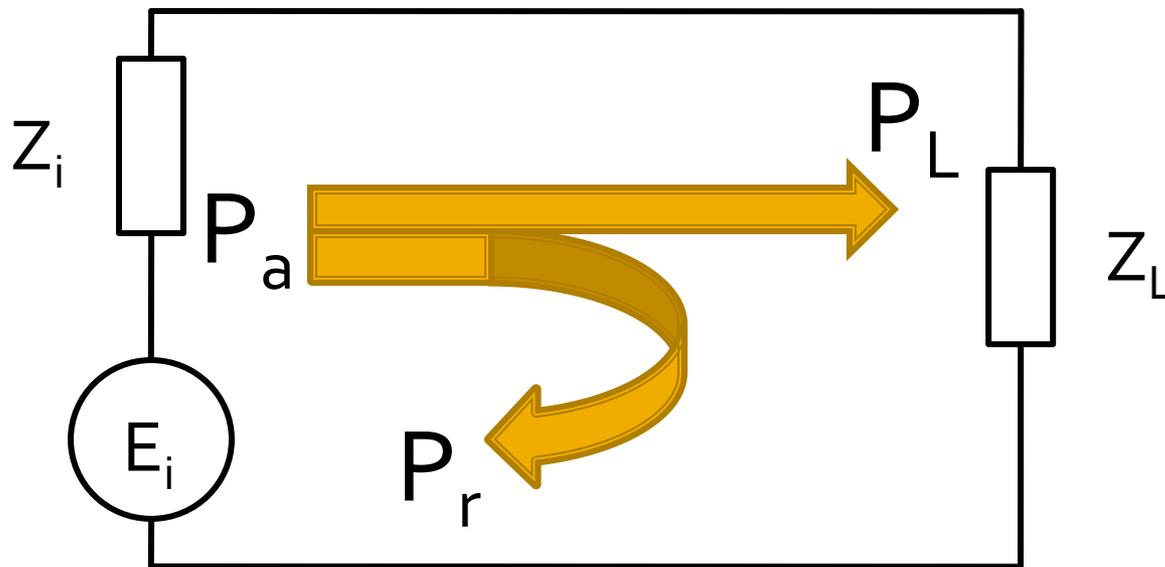
Any complex impedance,
named reference impedance

- Total voltage and current in terms of the power wave amplitudes

$$V = \frac{Z_R^* \cdot a + Z_R \cdot b}{\sqrt{R_R}}$$

$$I = \frac{a - b}{\sqrt{R_R}}$$

Reflection and power / Model – L3



$$P_a = \frac{|E_i|^2}{4R_i}$$

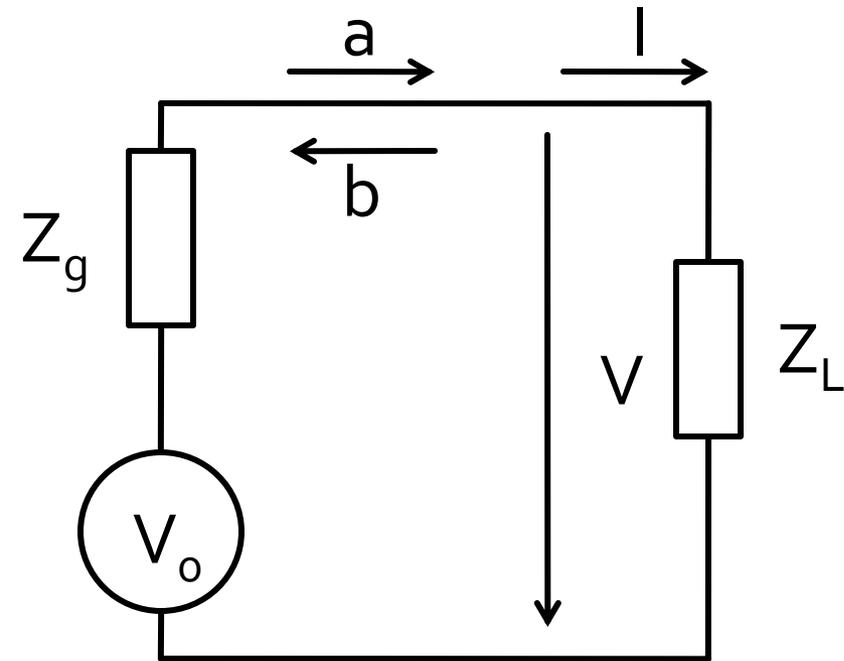
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0}$$

$$P_r = \frac{|E_i|^2}{4R_i} \cdot \left[\frac{(R_i - R_L)^2 + (X_i + X_L)^2}{(R_i + R_L)^2 + (X_i + X_L)^2} \right] = P_a \cdot |\Gamma|^2$$

- Γ , power reflection coefficient

Power waves



$$P_L = \frac{1}{2} \cdot \text{Re}\{V \cdot I^*\}$$

$$P_L = \frac{1}{2} \cdot \text{Re}\left\{ \frac{Z_R^* \cdot a + Z_R \cdot b}{\sqrt{R_R}} \cdot \left(\frac{a-b}{\sqrt{R_R}} \right)^* \right\}$$

$$P_L = \frac{1}{2R_R} \cdot \text{Re}\left\{ Z_R^* \cdot |a|^2 - Z_R^* \cdot a \cdot b^* + Z_R \cdot a^* \cdot b - Z_R \cdot |b|^2 \right\}$$

$$P_L = \frac{1}{2} \cdot |a|^2 - \frac{1}{2} \cdot |b|^2 \quad \underbrace{(z - z^*)}_{(z - z^*) = \text{Im}} \quad \boxed{\forall Z_R \in \mathbb{C}}$$

$$\Gamma_p = \frac{b}{a} = \frac{V - Z_R^* \cdot I}{V + Z_R \cdot I} = \frac{Z_L - Z_R^*}{Z_L + Z_R}$$

Power waves

$$V = \frac{V_0 \cdot Z_L}{Z_g + Z_L} \quad I = \frac{V_0}{Z_g + Z_L} \quad P_L = \frac{V_0^2}{2} \cdot \frac{R_L}{|Z_g + Z_L|^2}$$

- If we choose $Z_R = Z_L^*$

$$a = \frac{V + Z_R \cdot I}{2 \cdot \sqrt{R_R}} = V_0 \cdot \frac{\frac{Z_L}{Z_g + Z_L} + \frac{Z_L^*}{Z_g + Z_L}}{2 \cdot \sqrt{R_L}} = V_0 \cdot \frac{\sqrt{R_L}}{Z_g + Z_L}$$

$$b = \frac{V - Z_R^* \cdot I}{2 \cdot \sqrt{R_R}} = V_0 \cdot \frac{\frac{Z_L}{Z_g + Z_L} - \frac{Z_L}{Z_g + Z_L}}{2 \cdot \sqrt{R_L}} = 0$$

$$P_L = \frac{1}{2} \cdot |a|^2 = \frac{V_0^2}{2} \cdot \frac{R_L}{|Z_g + Z_L|^2}$$

Power waves

- When the load is conjugately matched to the generator

$$Z_g = Z_L^* \quad P_{L_{\max}} = \frac{1}{2} \cdot |a|^2 = \frac{V_0^2}{8 \cdot R_L}$$

- Power reflection: L3

$$Z_L = Z_i^* \quad P_{L_{\max}} \equiv P_a \quad \Gamma = \frac{Z - Z_0^*}{Z + Z_0^*}$$

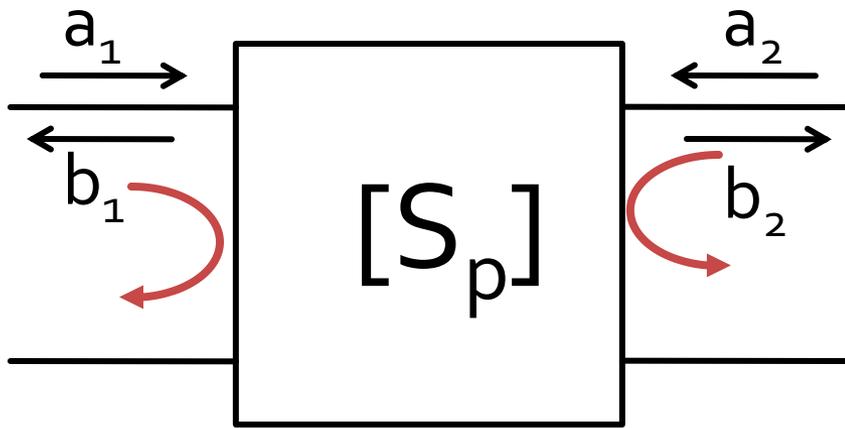
$$Z_L \neq Z_i^* \quad P_r = P_a \cdot |\Gamma|^2 \quad P_L = P_a - P_r = P_a - P_a \cdot |\Gamma|^2 = P_a \cdot (1 - |\Gamma|^2)$$

- Power reflection: L5

$$P_{L_{\max}} \equiv P_a = \frac{1}{2} \cdot |a|^2 \quad P_L = \frac{1}{2} \cdot |a|^2 - \frac{1}{2} \cdot |b|^2 \quad \Gamma_p = \frac{b}{a} = \frac{V - Z_R^* \cdot I}{V + Z_R \cdot I} = \frac{Z_L - Z_R^*}{Z_L + Z_R}$$

$$P_L = \frac{1}{2} \cdot |a|^2 - \frac{1}{2} \cdot |a|^2 \cdot |\Gamma_p|^2 \quad P_L = P_a \cdot (1 - |\Gamma_p|^2) \quad P_r = P_a \cdot |\Gamma_p|^2 = \frac{1}{2} \cdot |b|^2$$

Scattering matrix for power waves



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S'_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad S'_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

$$[b] = [S_p] \cdot [a]$$

Power waves

- To define the scattering matrix for power waves for an N-port network

$$[Z_R] = \begin{bmatrix} Z_{R1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Z_{Rn} \end{bmatrix} \quad [F] = \begin{bmatrix} 1/2\sqrt{R_{R1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/2\sqrt{R_{Rn}} \end{bmatrix}$$

$$[a] = [F] \cdot ([V] + [Z_R] \cdot [I])$$

$$[b] = [F] \cdot ([V] - [Z_R]^* \cdot [I])$$

$$[Z] \cdot [I] = [V]$$

Power waves for N ports

$$[b] = [F] \cdot ([Z] - [Z_R]^*) \cdot ([Z] + [Z_R])^{-1} \cdot [F]^{-1} \cdot [a]$$

- The scattering matrix for power waves, $[S_p]$

$$[b] = [S_p] \cdot [a]$$

$$[S_p] = [F] \cdot ([Z] - [Z_R]^*) \cdot ([Z] + [Z_R])^{-1} \cdot [F]^{-1}$$

- But: $[S] = ([Z] - [Z_0]) \cdot ([Z] + [Z_0])^{-1}$

- Typically

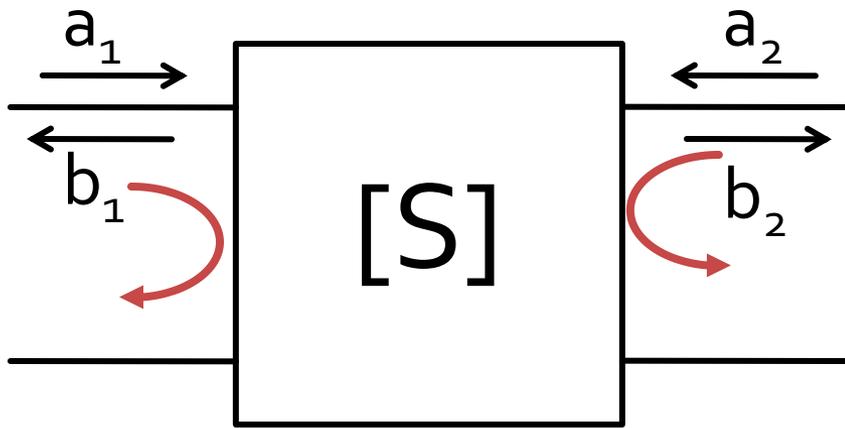
$$Z_{0i} = Z_{Ri} = R_0, \forall i$$

$$R_0 = 50\Omega$$

$$[S_p] \equiv [S]$$

- they coincide!!!

Scattering matrix – S

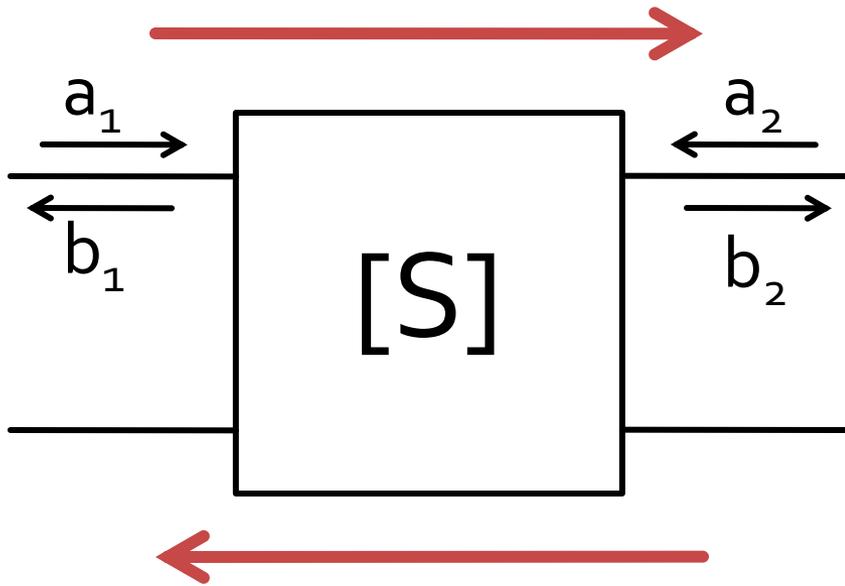


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

- S_{11} and S_{22} are **reflection coefficients** at ports 1 and 2 when the other port is **matched**

Scattering matrix – S

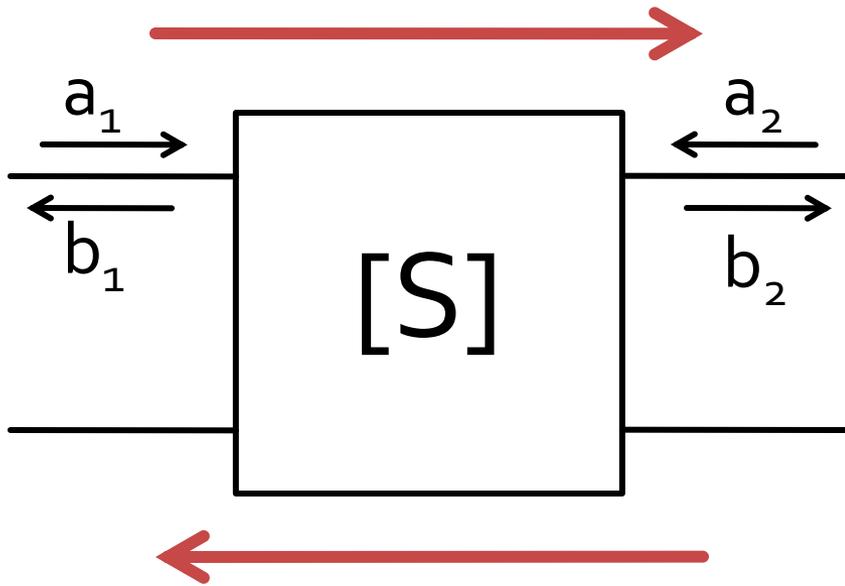


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \quad S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

- S_{21} si S_{12} are signal amplitude **gain** when the other port is **matched**

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{\text{Power in } Z_0 \text{ load}}{\text{Power from } Z_0 \text{ source}}$$

- a, b
 - information about signal power **AND** signal phase
- S_{ij}
 - network effect (gain) over signal power **including** phase information

Measuring S parameters - VNA

- Vector Network Analyzer

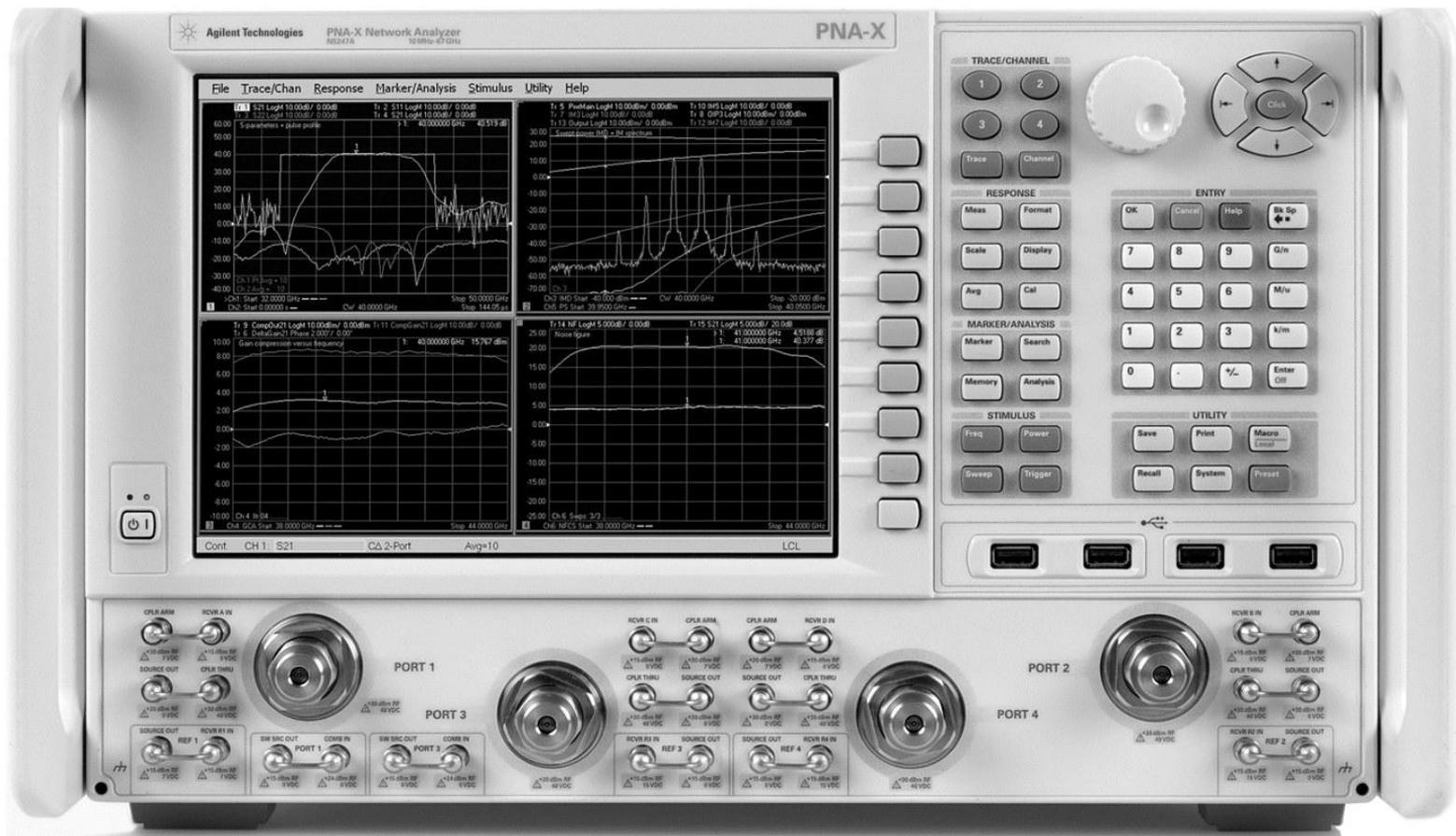


Figure 4.7
Courtesy of Agilent Technologies

Relation between two port S parameters and ABCD parameters

$$A = \sqrt{\frac{Z_{01}}{Z_{02}}} \frac{(1 + S_{11} - S_{22} - \Delta S)}{2S_{21}}$$

$$B = \sqrt{Z_{01}Z_{02}} \frac{(1 + S_{11} + S_{22} + \Delta S)}{2S_{21}}$$

$$C = \frac{1}{\sqrt{Z_{01}Z_{02}}} \frac{1 - S_{11} - S_{22} + \Delta S}{2S_{21}}$$

$$D = \sqrt{\frac{Z_{02}}{Z_{01}}} \frac{1 - S_{11} + S_{22} - \Delta S}{2S_{21}}$$

$$\Delta S = S_{11}S_{22} - S_{12}S_{21}$$

$$S_{11} = \frac{AZ_{02} + B - CZ_{01}Z_{02} - DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{12} = \frac{2(AD - BC)\sqrt{Z_{01}Z_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

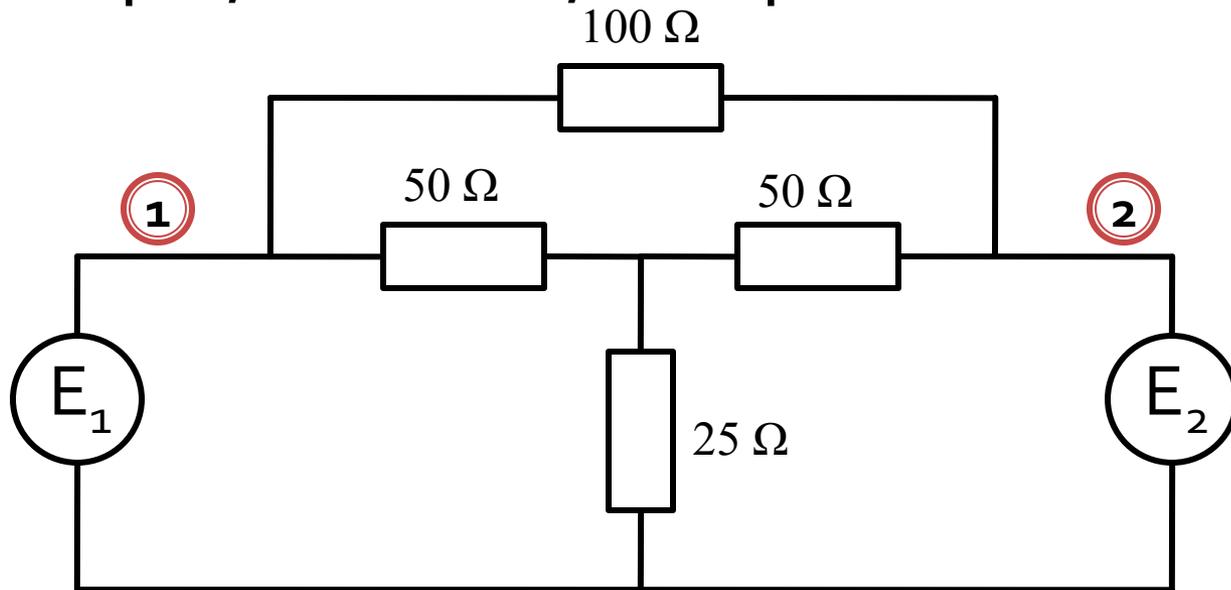
$$S_{21} = \frac{2\sqrt{Z_{01}Z_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{22} = \frac{-AZ_{02} + B - CZ_{01}Z_{02} + DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

Even/Odd Mode Analysis

Even/Odd Mode Analysis

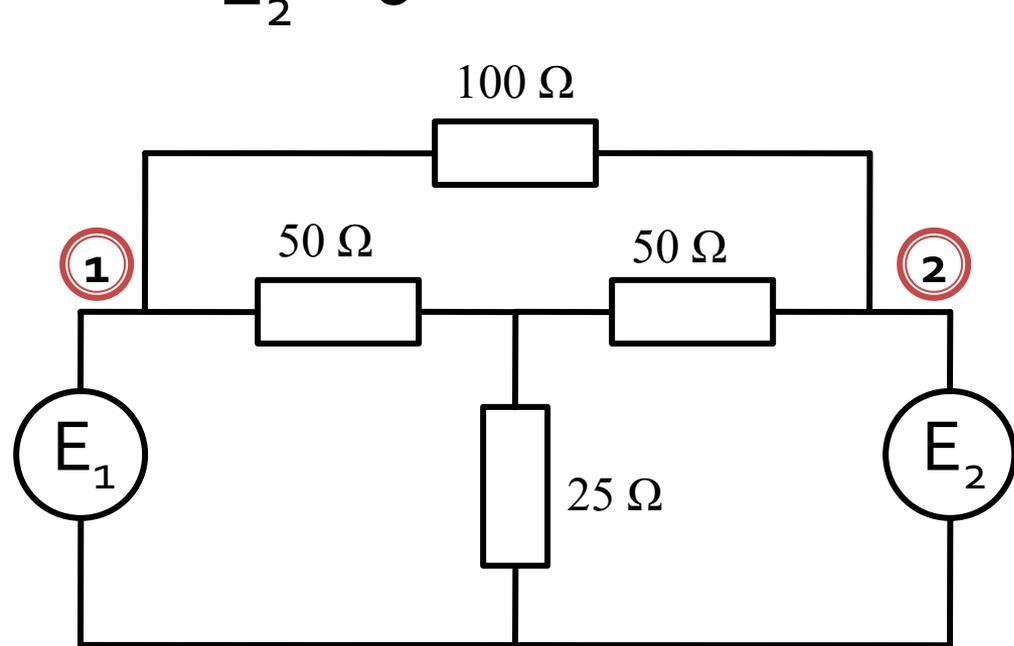
- useful method, necessary even for multiple ports
- example, resistors, two port circuit



Even/Odd Mode Analysis

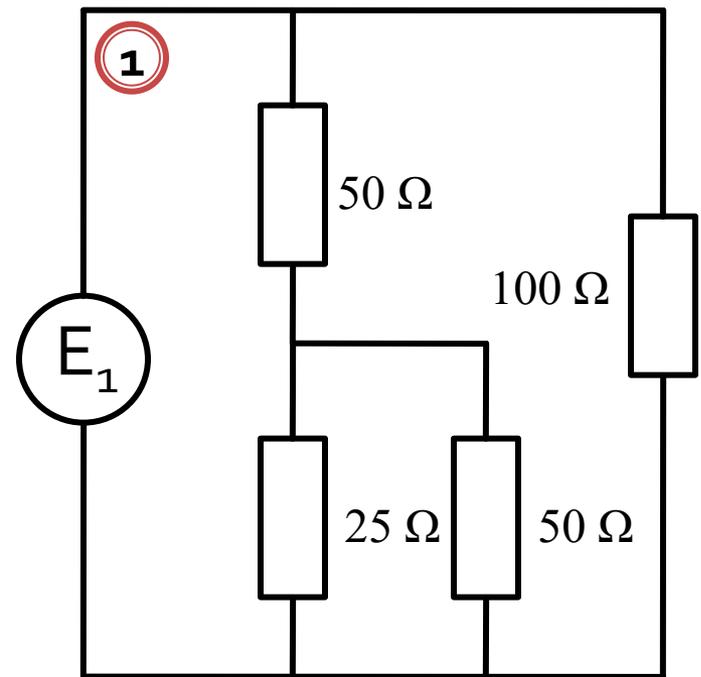
- assume we want to compute Y_{11}
- $E_2 = 0$

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}$$



$$R_{ech} = 100\Omega \parallel (50\Omega + 25\Omega \parallel 50\Omega) =$$

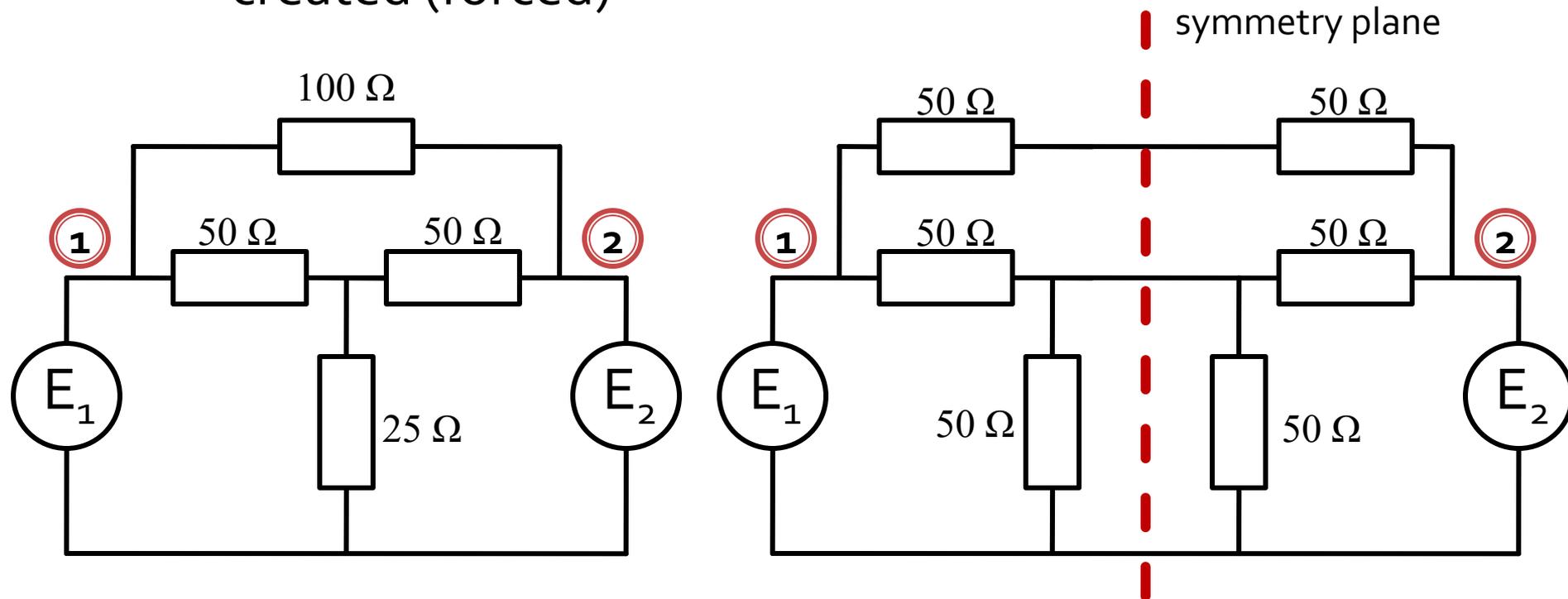
$$= 100\Omega \parallel (50\Omega + 16.67\Omega) = 100\Omega \parallel 66.67\Omega = 40\Omega$$



$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = 0.025S$$

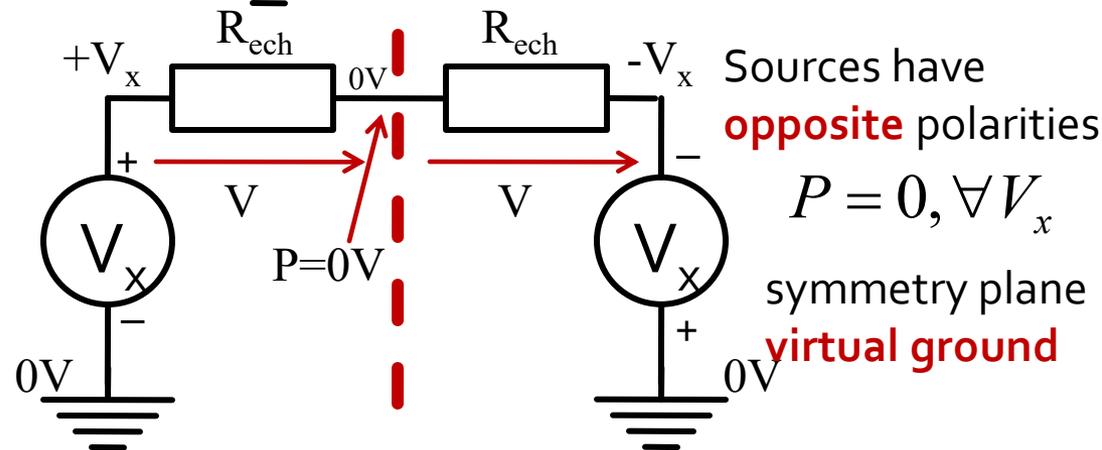
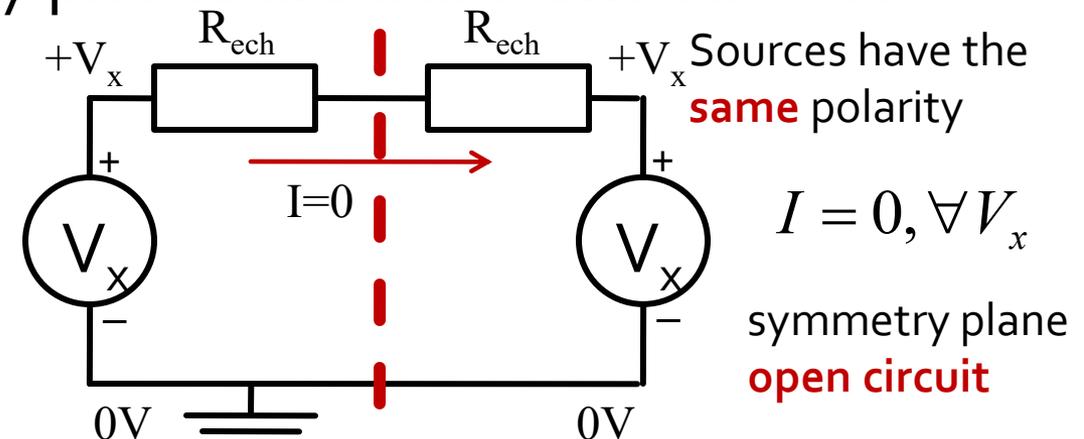
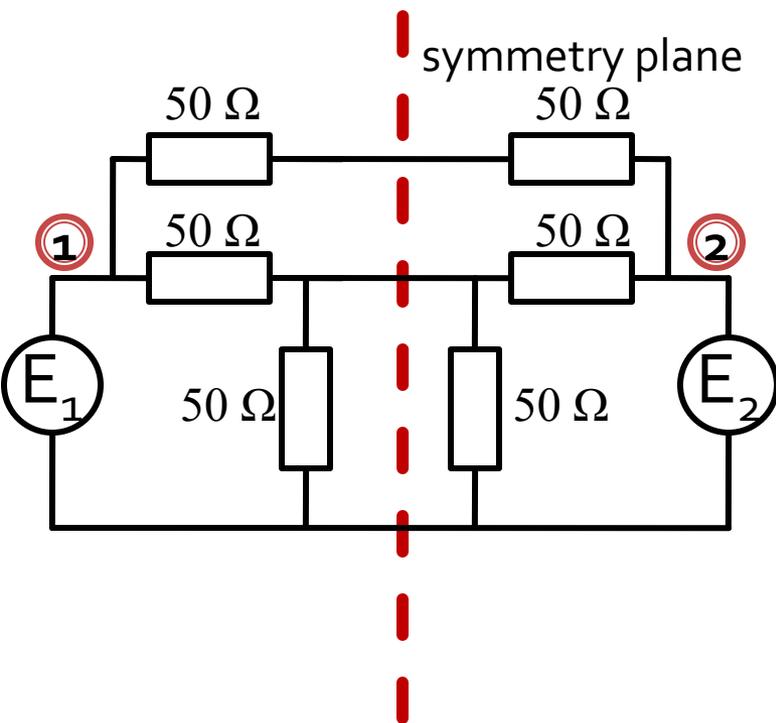
Even/Odd Mode Analysis

- Even/Odd mode analysis benefit from the existence of symmetry planes in the circuit
 - existing or
 - created (forced)



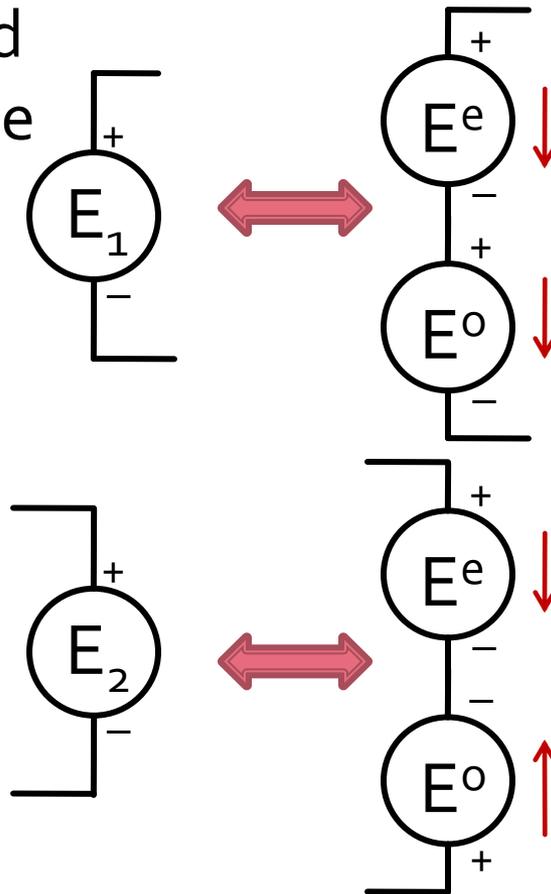
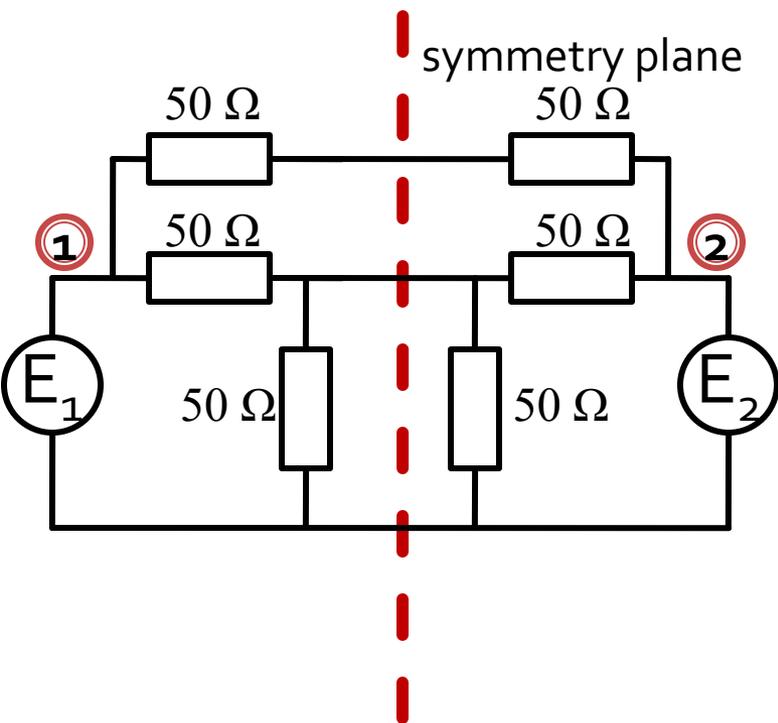
Even/Odd Mode Analysis

- when exciting the ports with symmetric/anti-symmetric sources the symmetry planes are transformed into:
 - open circuit
 - virtual ground



Even/Odd Mode Analysis

- the combination of any two sources is equivalent for linear circuits with the superposition of:
 - a symmetric source and
 - a anti-symmetric source



$$E_1 = E^e + E^o$$

$$E_2 = E^e - E^o$$

$$E^e = \frac{E_1 + E_2}{2}$$

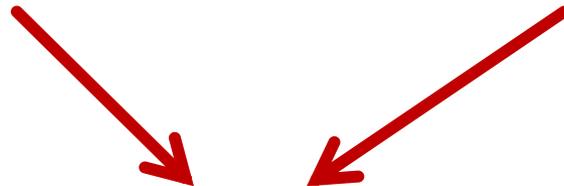
$$E^o = \frac{E_1 - E_2}{2}$$

Even/Odd Mode Analysis

- In linear circuits the **superposition principle** is always true
 - the response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually

$$\begin{aligned} \text{Response (Source1 + Source2)} &= \\ &= \text{Response (Source1)} + \text{Response (Source2)} \end{aligned}$$

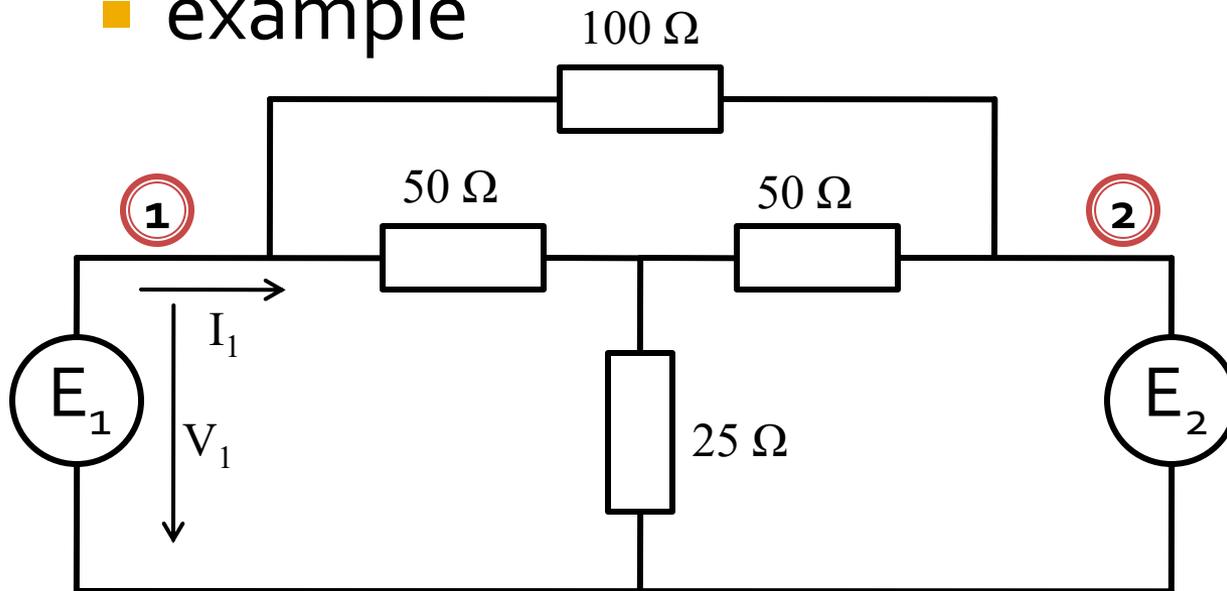
$$\text{Response (ODD + EVEN)} = \text{Response (ODD)} + \text{Response (EVEN)}$$



We can benefit from existing symmetries !!

Even/Odd Mode Analysis

■ example

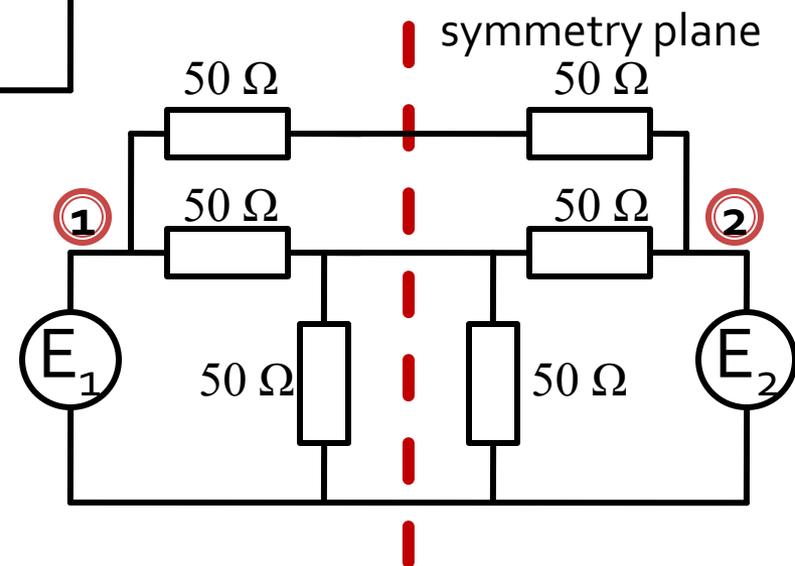


$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

$$V_2 \equiv E_2 = 0 \Rightarrow$$

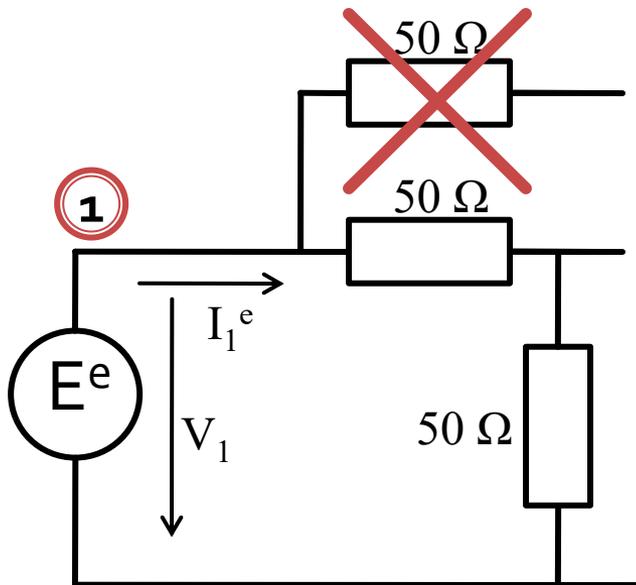
$$E^e = \frac{E_1}{2}$$

$$E^o = \frac{E_1}{2}$$



Even/Odd Mode Analysis

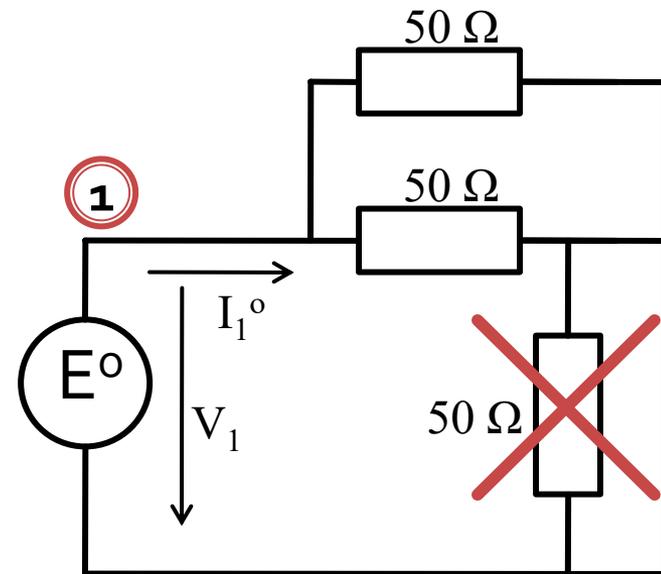
- Even/Odd mode analysis



$$R_{ech}^e = 50\Omega + 50\Omega = 100\Omega$$

$$I_1^e = \frac{E^e}{R_{ech}^e} = \frac{E_1/2}{100\Omega} = \frac{E_1}{200\Omega}$$

EVEN → symmetry plane **open circuit**



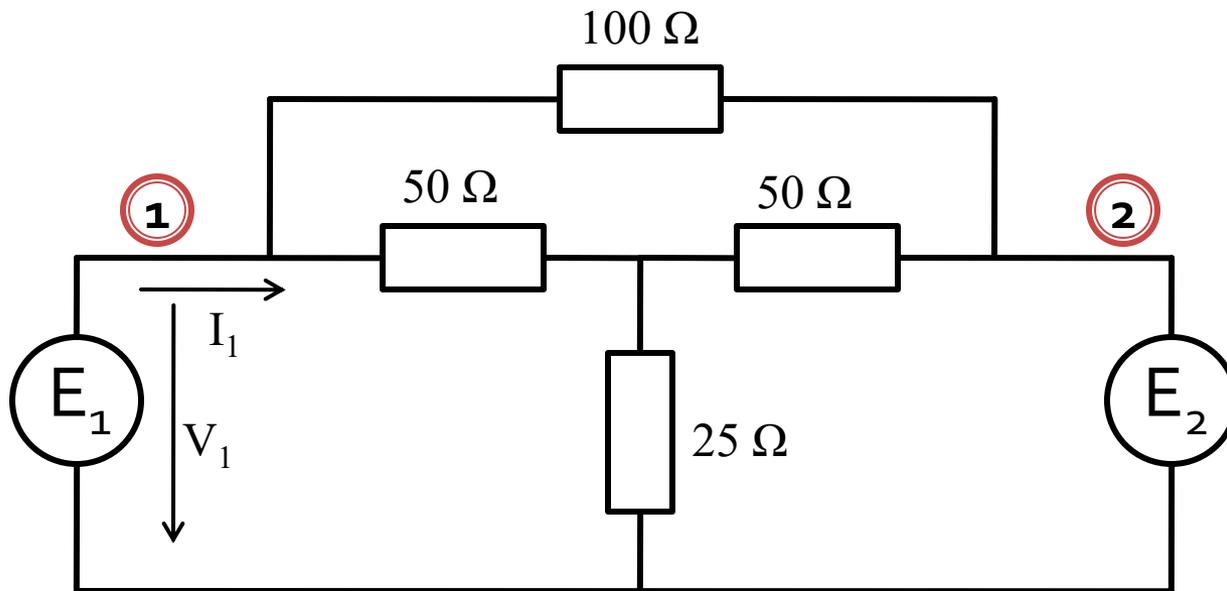
$$R_{ech}^o = 50\Omega || 50\Omega = 25\Omega$$

$$I_1^o = \frac{E^o}{R_{ech}^o} = \frac{E_1/2}{25\Omega} = \frac{E_1}{50\Omega}$$

ODD → symmetry plane **virtual ground**

Even/Odd Mode Analysis

- superposition principle



$$I_1 = I_1^e + I_1^o$$

$$V_1 = V_1^e + V_1^o$$

$$I_1 = I_1^e + I_1^o = \frac{E_1}{200\Omega} + \frac{E_1}{50\Omega} = \frac{E_1}{40\Omega}$$

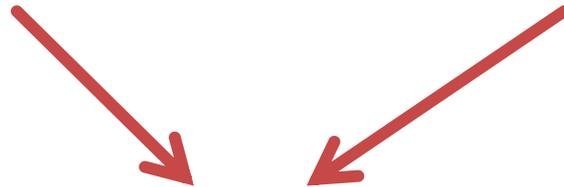
$$V_1 = V_1^e + V_1^o = E_1$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{40\Omega} = 0.025S$$

Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
 - reduction of the circuit complexity
 - decrease of the **number of ports** (**main** advantage)

$$\text{Response (ODD + EVEN)} = \text{Response (ODD)} + \text{Response (EVEN)}$$



We can benefit from existing symmetries !!

Power dividers and directional couplers

Course Topics

- Transmission lines
- Impedance matching and tuning
- **Directional couplers**
- **Power dividers**
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~

Introduction

Power dividers and couplers

- Desired functionality:
 - division
 - combining
- of signal power

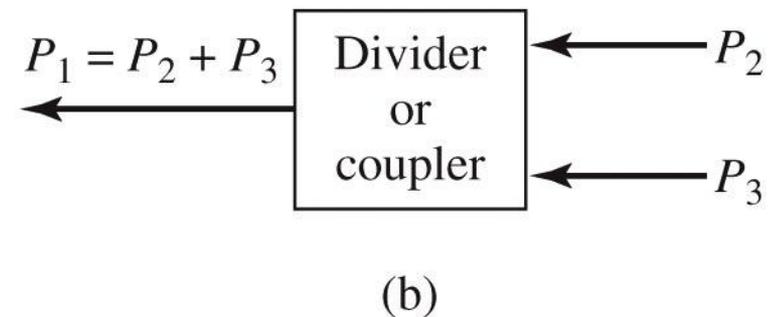
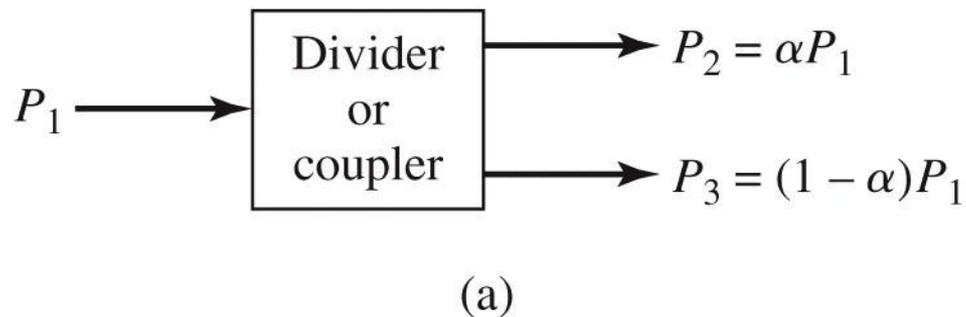
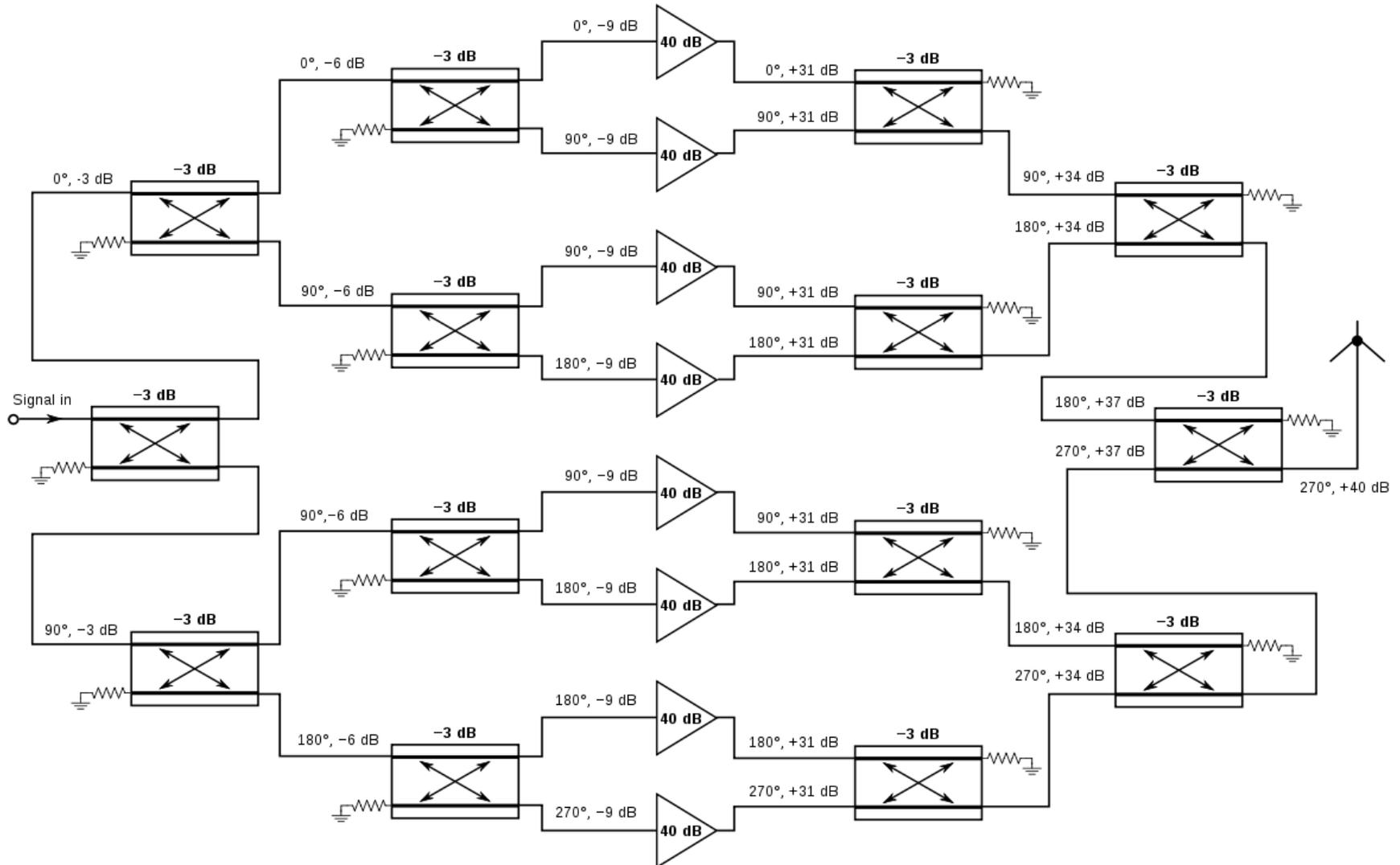


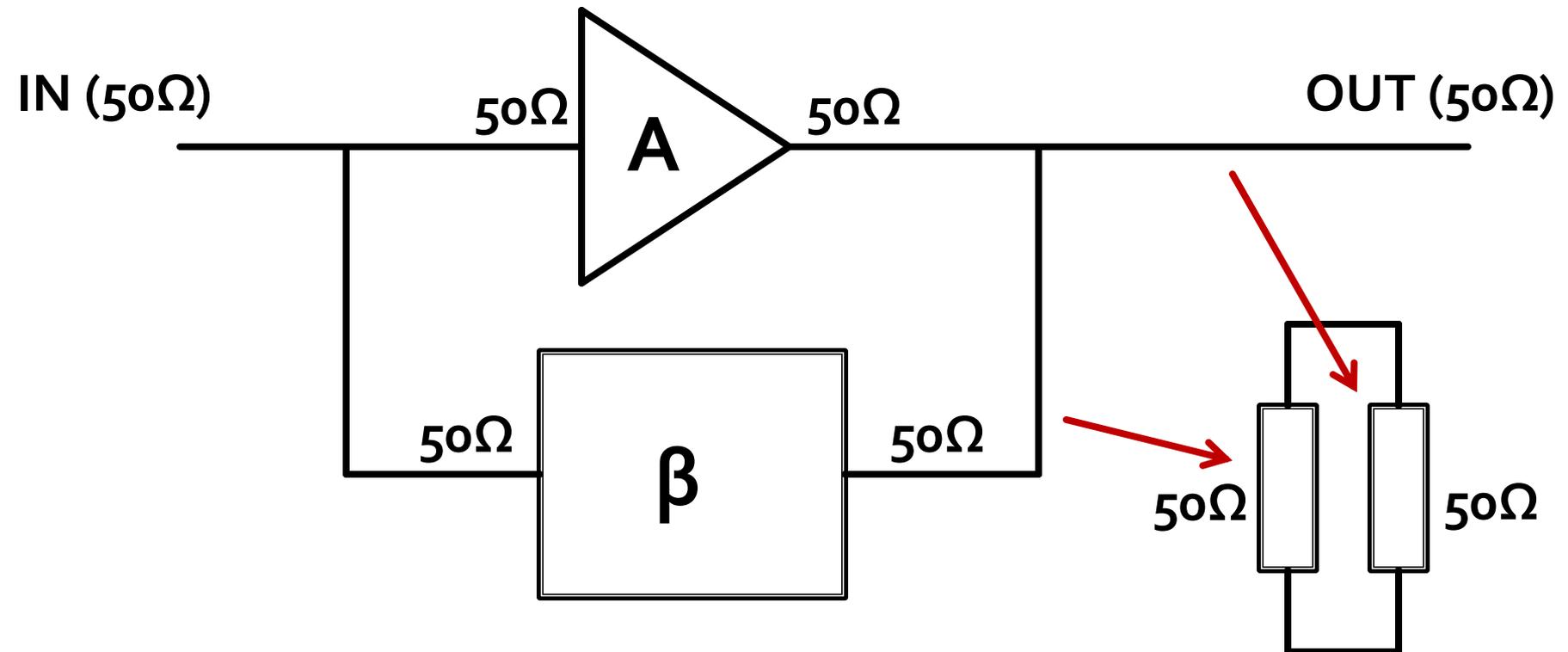
Figure 7.1
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Balanced amplifiers



Matching

- feedback amplifier



Three-Port Networks

- also known as T-Junctions
- characterized by a 3×3 **S** matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
 - anisotropic materials (usually ferrites)
 - active circuits
- to avoid power loss, we would like to have a network that is:
 - **lossless**, and
 - **matched at all ports**
 - to avoid reflection power “loss”

Three-Port Networks

- reciprocal

$$[S] = [S]^t \quad S_{ij} = S_{ji}, \forall j \neq i$$

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{23} = S_{32}$$

- matched at all ports

$$S_{ii} = 0, \forall i \quad S_{11} = 0, S_{22} = 0, S_{33} = 0$$

- then the S matrix is:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Three-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- lossless network

- all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \quad \sum_{k=1}^N S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^N S_{ki} \cdot S_{ki}^* = 1 \quad \sum_{k=1}^N S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Three-Port Networks

- lossless network

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

$$\sum_{k=1}^N S_{ki} \cdot S_{ki}^* = 1$$

$$\sum_{k=1}^N S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

- 6 equations / 3 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad S_{13}^* S_{23} = 0$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \quad S_{12}^* S_{13} = 0$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \quad S_{23}^* S_{12} = 0$$

- **no solution** is possible

Three-Port Networks

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
 - no solution is possible
- A three-port network **cannot** be simultaneously:
 - reciprocal
 - lossless
 - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Nonreciprocal Three-Port Networks

- usually containing anisotropic materials, ferrites
- **nonreciprocal**, but matched at all ports and lossless $S_{ij} \neq S_{ji}$

- S matrix

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

- 6 equations / 6 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad S_{31}^* S_{32} = 0$$

$$|S_{21}|^2 + |S_{23}|^2 = 1 \quad S_{21}^* S_{23} = 0$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \quad S_{12}^* S_{13} = 0$$

Nonreciprocal Three-Port Networks

- two possible solutions
- circulators
 - clockwise circulation

$$S_{12} = S_{23} = S_{31} = 0$$

$$|S_{21}| = |S_{32}| = |S_{13}| = 1$$

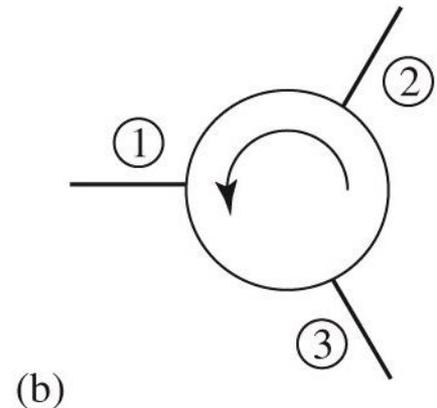
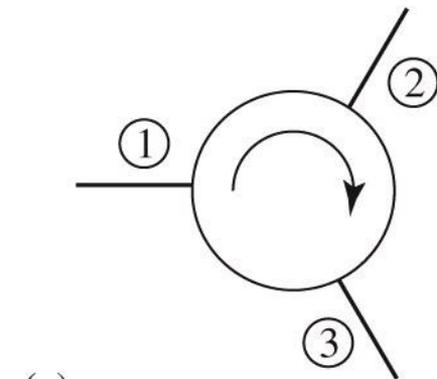
$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- counterclockwise circulation

$$S_{21} = S_{32} = S_{13} = 0$$

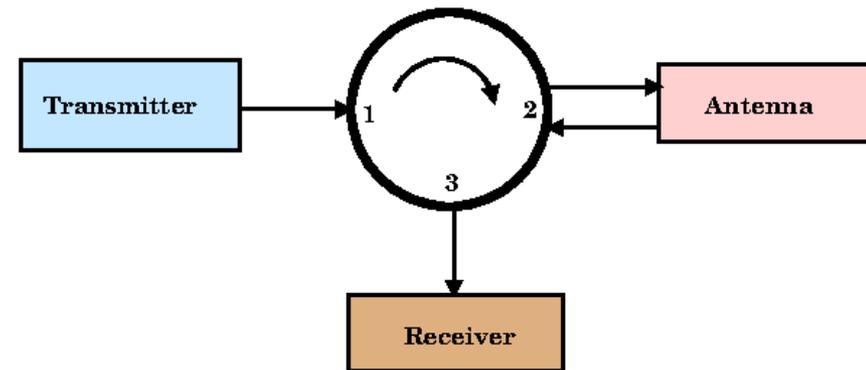
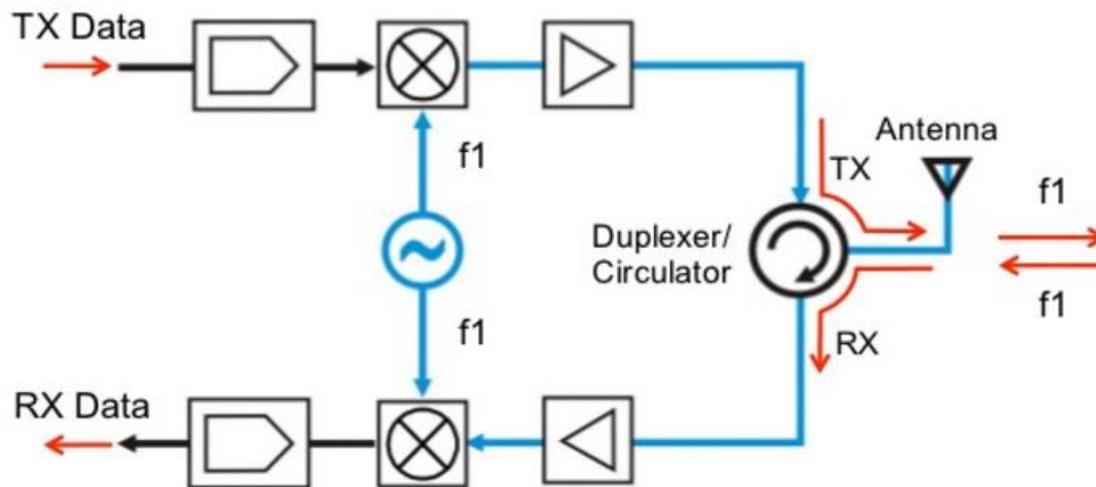
$$|S_{12}| = |S_{23}| = |S_{31}| = 1$$

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



Nonreciprocal Three-Port Networks

- circulator often found in duplexer



Mismatched Three-Port Networks

- A lossless and reciprocal three-port network can be matched only on two ports, eg. 1 and 2:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}^* S_{23} = 0$$

$$S_{12}^* S_{13} + S_{23}^* S_{33} = 0$$

$$S_{23}^* S_{12} + S_{33}^* S_{13} = 0$$

$$S_{13} = S_{23} = 0$$

$$|S_{13}| = |S_{23}|$$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{12}|^2 + |S_{23}|^2 = 1$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1$$

$$|S_{12}| = |S_{33}| = 1$$

Mismatched Three-Port Networks

- A lossless and reciprocal three-port network

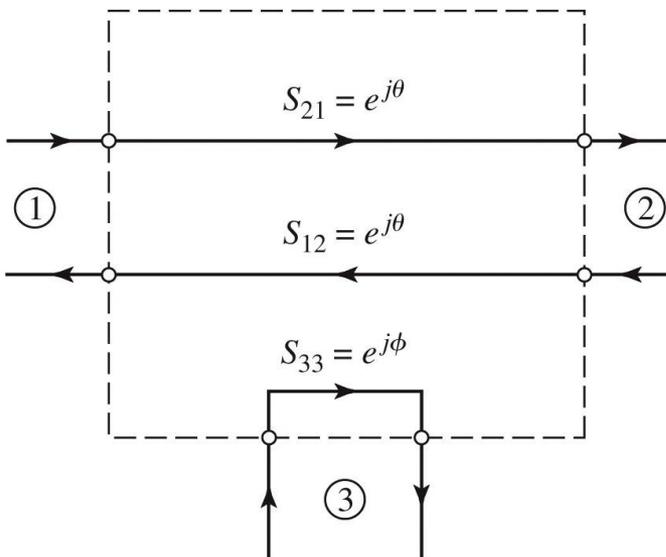
$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13} = S_{23} = 0 \quad |S_{12}| = |S_{33}| = 1$$

$$S_{12} = e^{j\theta}$$

$$S_{33} = e^{j\phi}$$

$$[S] = \begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$



- A lossless and reciprocal three-port network **degenerates** into two separate components:

- a matched two-port **line**
- a totally **mismatched one-port**:

Four-Port Networks

- characterized by a 4×4 **S** matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
 - anisotropic materials (usually ferrites)
 - active circuits
- to avoid power loss, we would like to have a network that is:
 - **lossless**, and
 - **matched at all ports**
 - to avoid reflection power “loss”

Four-Port Networks

- reciprocal

$$[S] = [S]^t \quad S_{ij} = S_{ji}, \forall j \neq i$$

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{23} = S_{32}$$

- matched at all ports

$$S_{ii} = 0, \forall i \quad S_{11} = 0, S_{22} = 0, S_{33} = 0, S_{44} = 0$$

- then the S matrix is:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

Four-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

- lossless network

- all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \quad \sum_{k=1}^N S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^N S_{ki} \cdot S_{ki}^* = 1 \quad \sum_{k=1}^N S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Four-Port Networks

$$S_{13}^* \cdot S_{23} + S_{14}^* \cdot S_{24} = 0 \quad / \cdot S_{24}^*$$

$$S_{14}^* \cdot S_{13} + S_{24}^* \cdot S_{23} = 0 \quad / \cdot S_{13}^*$$

$$S_{14}^* \cdot (|S_{13}|^2 - |S_{24}|^2) = 0$$

$$S_{12}^* \cdot S_{23} + S_{14}^* \cdot S_{34} = 0 \quad / \cdot S_{12}^*$$

$$S_{14}^* \cdot S_{12} + S_{34}^* \cdot S_{23} = 0 \quad / \cdot S_{34}^*$$

$$S_{23} \cdot (|S_{12}|^2 - |S_{34}|^2) = 0$$

- one solution: $S_{14} = S_{23} = 0$
- resulting coupler is **directional**

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad \rightarrow \quad |S_{13}| = |S_{24}|$$

$$|S_{12}|^2 + |S_{24}|^2 = 1 \quad \rightarrow \quad |S_{13}| = |S_{24}|$$

$$|S_{13}|^2 + |S_{34}|^2 = 1 \quad \rightarrow \quad |S_{12}| = |S_{34}|$$

$$|S_{24}|^2 + |S_{34}|^2 = 1 \quad \rightarrow \quad |S_{12}| = |S_{34}|$$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

Four-Port Networks

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix} \quad |S_{12}| = |S_{34}| = \alpha \quad |S_{13}| = |S_{24}| = \beta$$

β – voltage coupling coefficient

- We can choose the phase reference

$$S_{12} = S_{34} = \alpha \quad S_{13} = \beta \cdot e^{j\theta} \quad S_{24} = \beta \cdot e^{j\phi}$$

$$S_{12}^* \cdot S_{13} + S_{24}^* \cdot S_{34} = 0 \quad \rightarrow \quad \theta + \phi = \pi \pm 2 \cdot n \cdot \pi$$

$$|S_{12}|^2 + |S_{24}|^2 = 1 \quad \rightarrow \quad \alpha^2 + \beta^2 = 1$$

- The other possible solution for previous equations offer either essentially the same result (with a different phase reference) or the degenerate case (2 separate two port networks side by side)

$$S_{14}^* \cdot (|S_{13}|^2 - |S_{24}|^2) = 0 \quad S_{23} \cdot (|S_{12}|^2 - |S_{34}|^2) = 0$$

Four-Port Networks

- A four-port network simultaneously:
 - matched at all ports
 - reciprocal
 - lossless
- is **always directional**
 - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S] = \begin{bmatrix} 0 & \alpha & \beta \cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta \cdot e^{j\phi} \\ \beta \cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta \cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Four-Port Networks

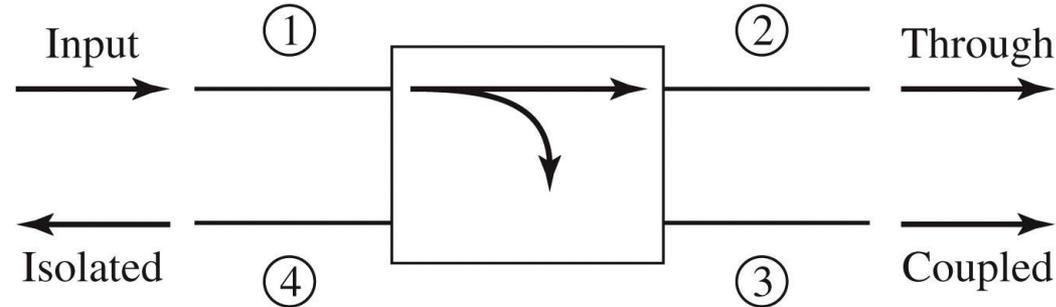
- two particular choices commonly occur in practice
 - A Symmetric Coupler (90°) $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

- An Antisymmetric Coupler (180°) $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Directional Coupler



$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Coupling

$$C = 10 \log \frac{P_1}{P_3} = -20 \cdot \log(\beta) [\text{dB}]$$

Directivity

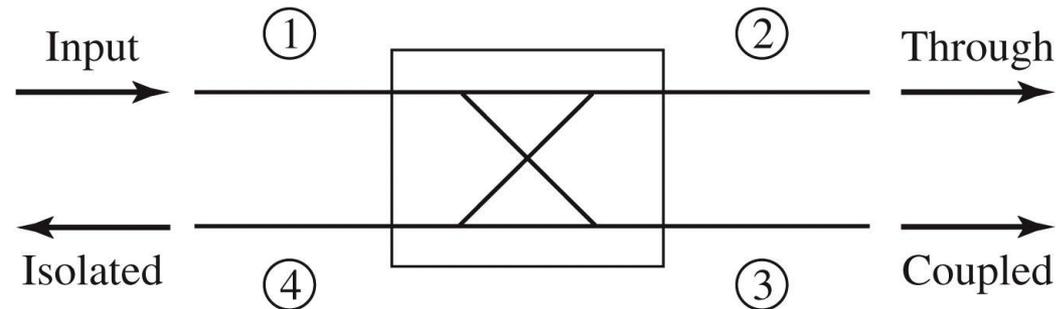
$$D = 10 \log \frac{P_3}{P_4} = 20 \cdot \log \left(\frac{\beta}{|S_{14}|} \right) [\text{dB}]$$

Isolation

$$I = 10 \log \frac{P_1}{P_4} = -20 \cdot \log |S_{14}| [\text{dB}]$$

$$I = D + C, \quad [\text{dB}]$$

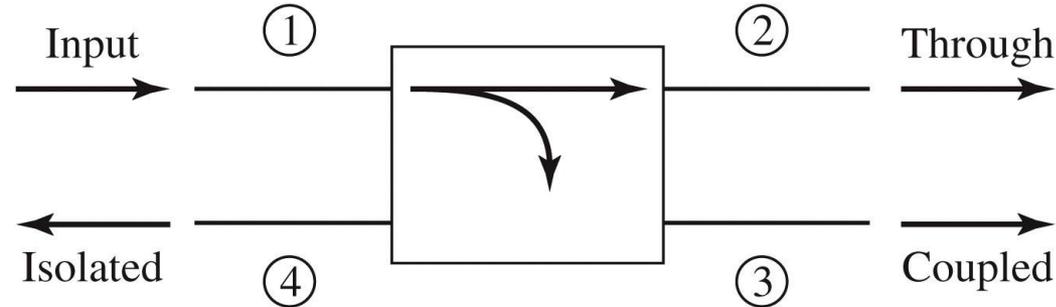
Figure 7.4
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Directional Couplers

Laboratory no. 2

Directional Coupler



$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Cuplaj

$$C = 10 \log \frac{P_1}{P_3} = -20 \cdot \log(\beta) [\text{dB}]$$

Directivitate

$$D = 10 \log \frac{P_3}{P_4} = 20 \cdot \log \left(\frac{\beta}{|S_{14}|} \right) [\text{dB}]$$

Izolare

$$I = 10 \log \frac{P_1}{P_4} = -20 \cdot \log |S_{14}| [\text{dB}]$$

$$I = D + C, \text{ dB}$$

Figure 7.4
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The quadrature (90°) hybrid

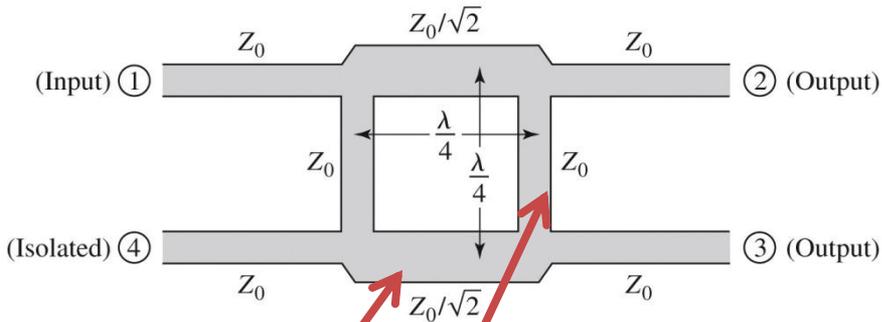


Figure 7.21
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$$y_2^2 = 1 + y_1^2$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$C[\text{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}$$

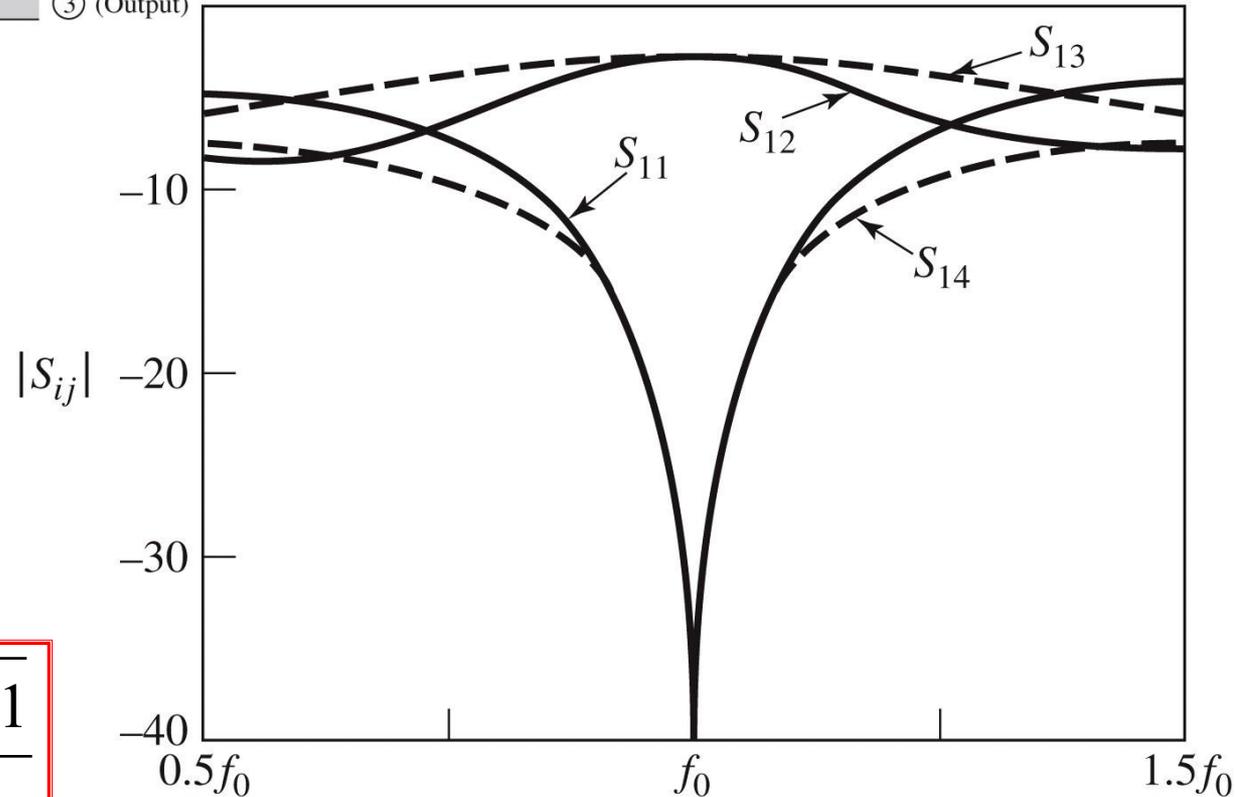
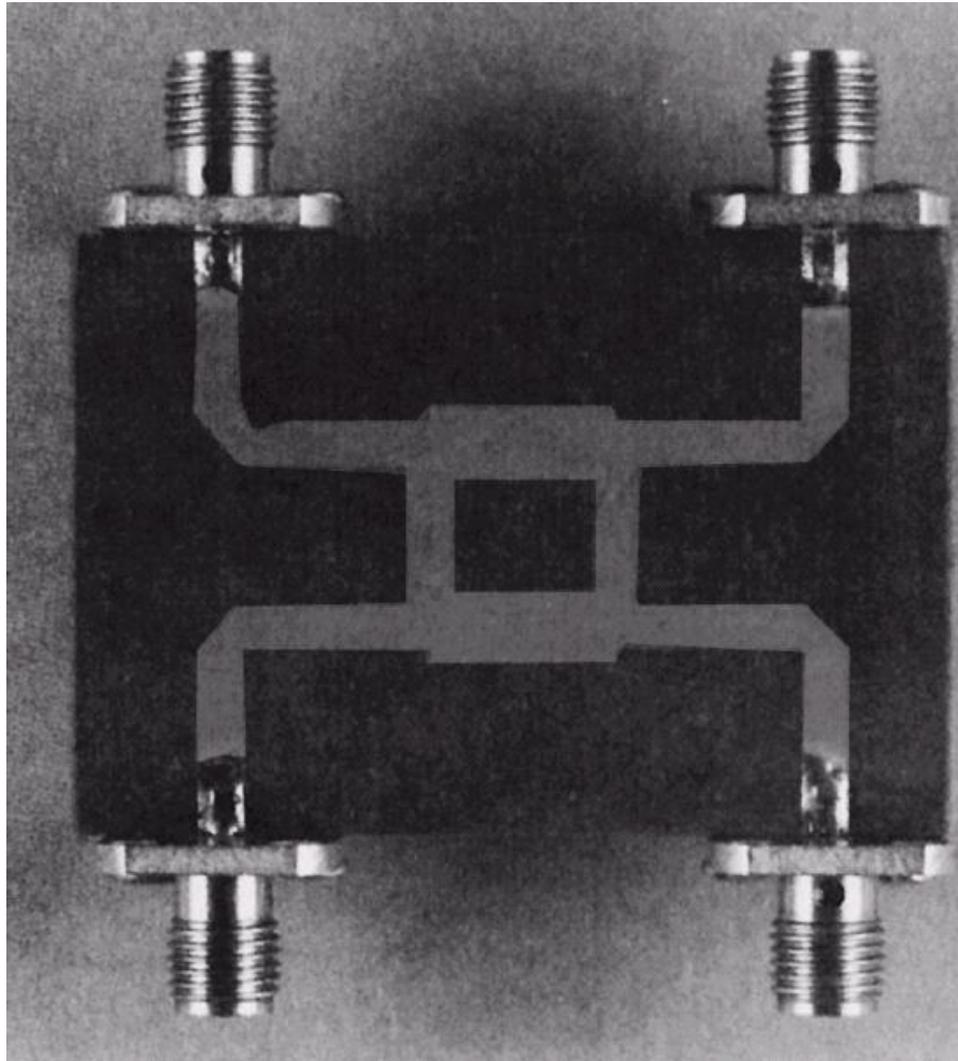
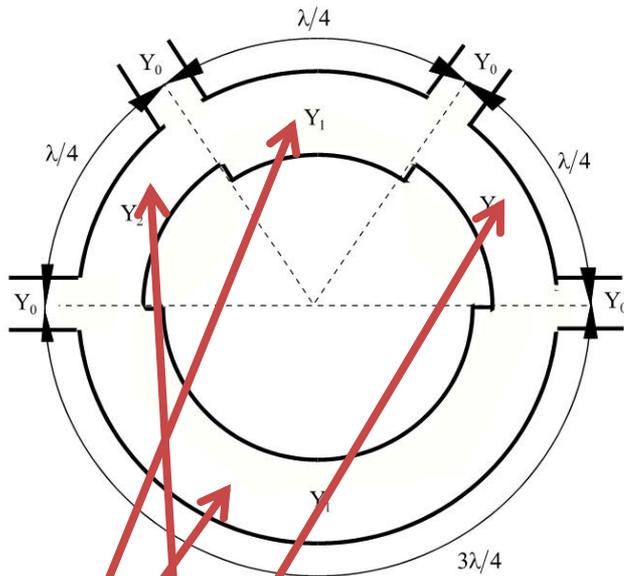


Figure 7.25
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Quadrature coupler



The 180° ring hybrid (rat-race)



$$y_1^2 + y_2^2 = 1$$

$$|\beta| = y_1$$

$$C \text{ [dB]} = -20 \cdot \log_{10}(y_1)$$

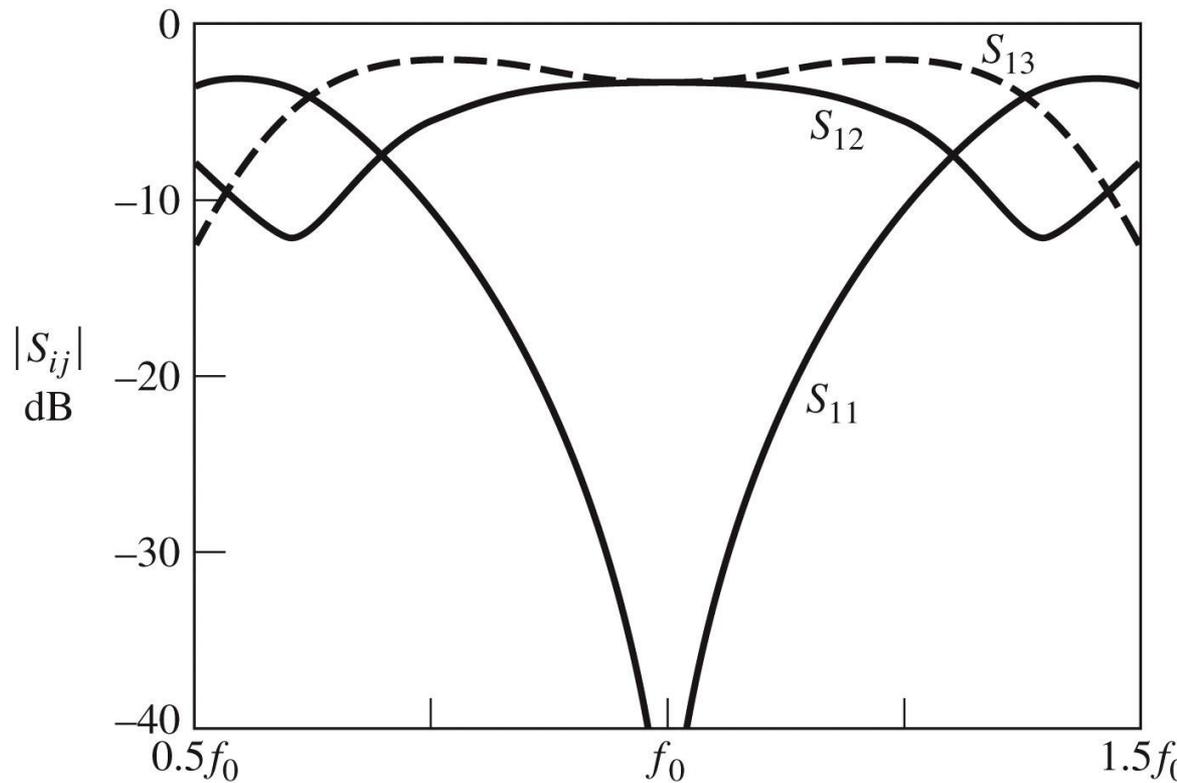


Figure 7.46
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Ring coupler

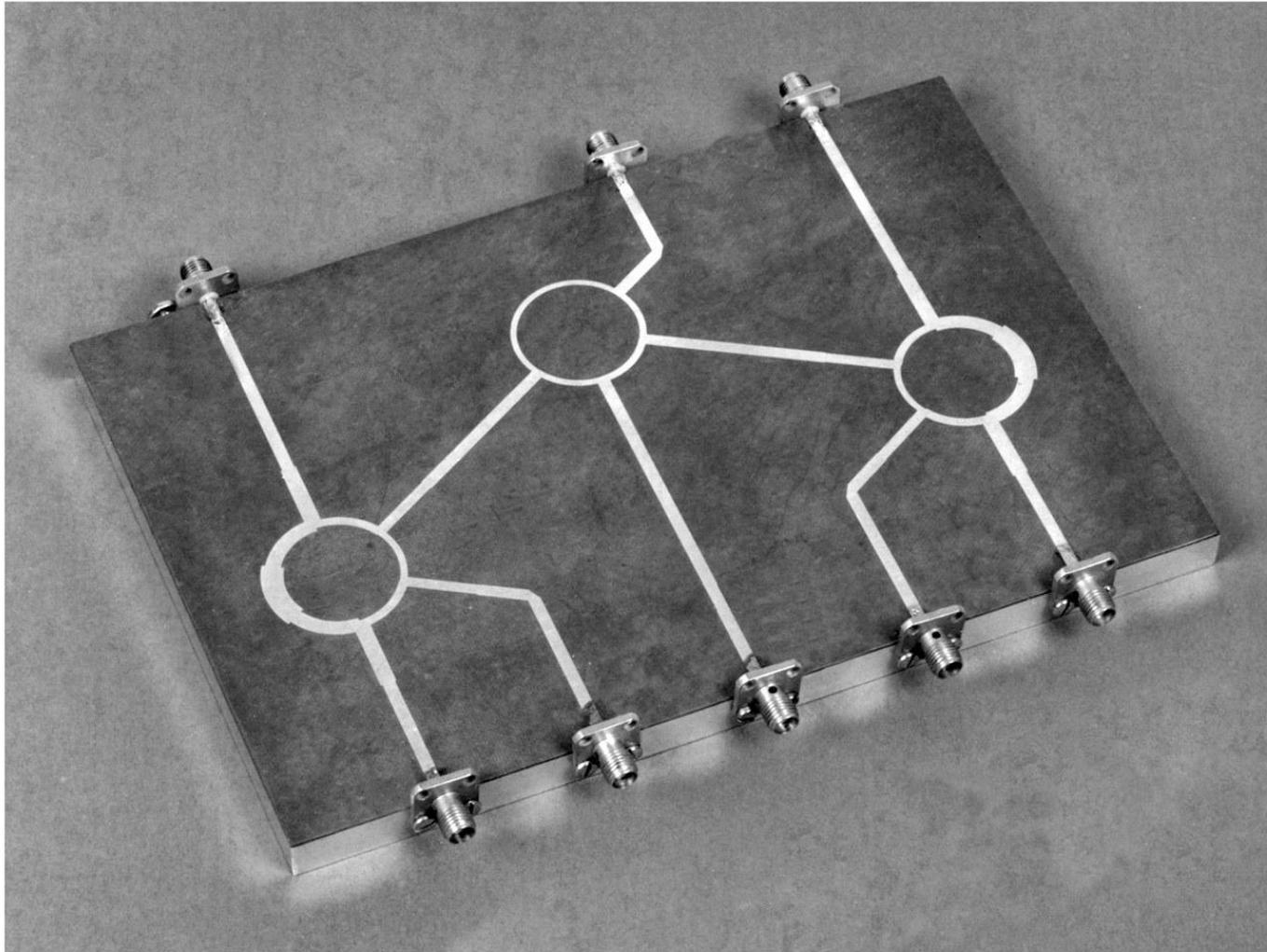
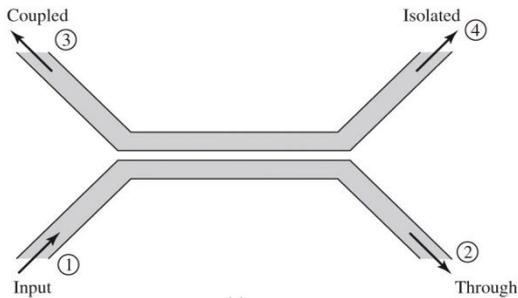


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled Line Coupler



Coupling, Directivity (dB)

$$Z_{ce} Z_{co} = Z_0^2$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$C [\text{dB}] = -20 \cdot \log_{10} \left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}} \right)$$

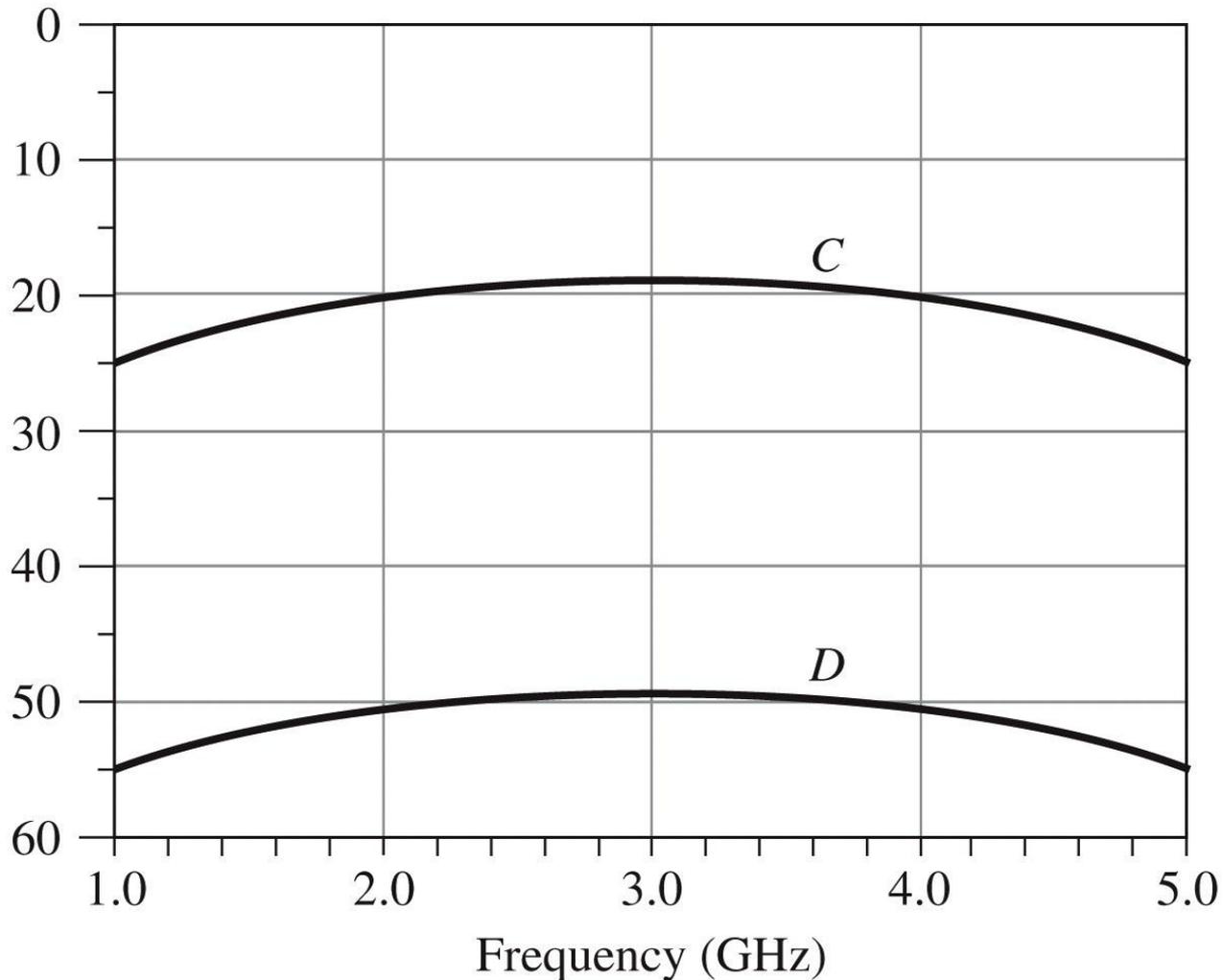
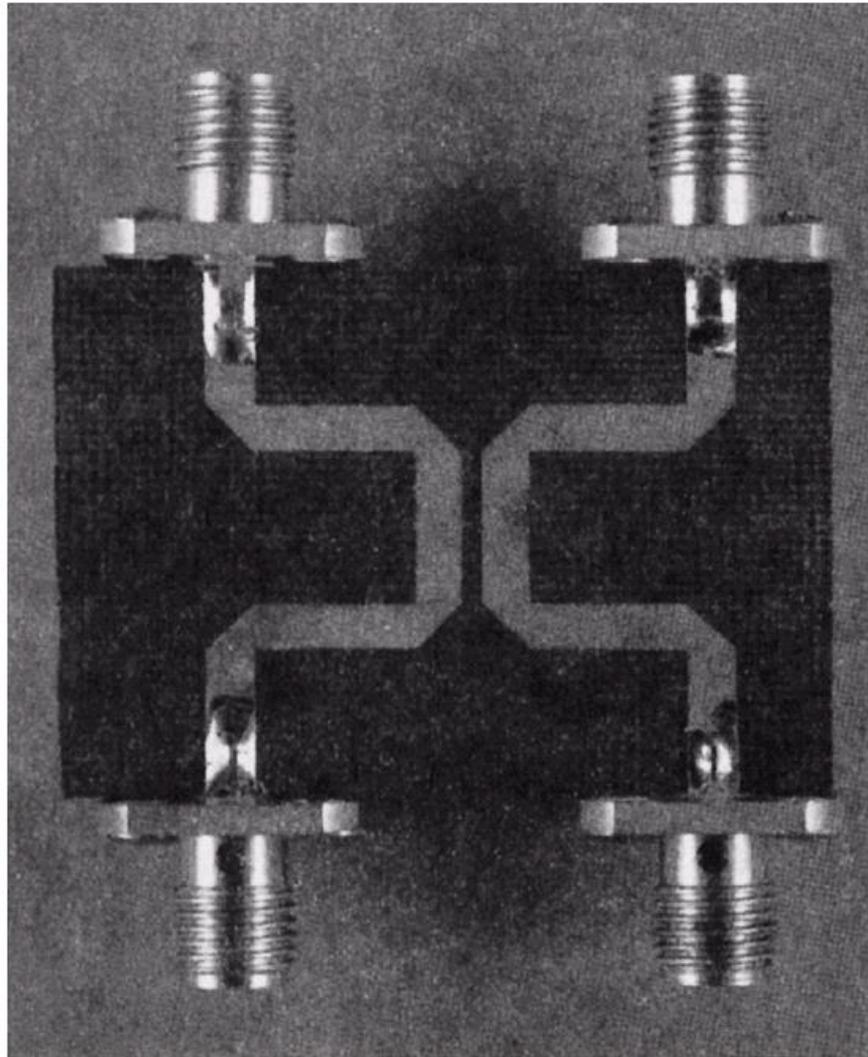


Figure 7.34
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Coupled line coupler



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